Welfare and Trade Margins with Multinational Production

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July 2017

Welfare and Trade Elasticity

with Multinational Production[*]

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Abstract

The presence of multinational production in an otherwise standard model of trade with heterogeneous firms generates an endogenous upper bound on the productivity distribution. In this setting, we show that trade elasticity is no longer constant, and depends on both supply and demand parameters. We isolate the component of welfare associated with multinational firms, and show that welfare gains differ with respect to models with only export. The model is then calibrated to analyze counterfactual scenarios. Multinational production with intra-firm trade increases welfare gains by up to 4 percent with respect to a model with only export and no truncation. Multinational production *à la* Helpman et al. (2004) generates the largest welfare gains from liberalization.

JEL classification: F10, F12, F23
Keywords: Multinational production, export, elasticity, welfare.

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1. INTRODUCTION

Engaging in international trade is an exceedingly rare activity: In 2000, only 4 percent of all U.S. firms were exporting (Bernard et al. (2007), Eaton et al. (2011) among others). In this context, multinational firms play a key role: 90 percent of U.S. exports and imports occur through them with 50 percent of U.S. imports taking place within the same firm rather than through arm's length (Bernard et al., 2009). Several empirical studies convey the idea that intra-firm trade is mainly related to the transfers of capabilities within the corporation. For example, Ramondo and Ruhl (2013) find that most U.S. foreign affiliates are not created for multistage production chains, but as outlets to produce and then supply in the local market.

We adopt the model of Irarrazabal et al. (2013) to account for multinational production with intra-firm trade, and assume perfect symmetry and free entry. In this framework, each foreign affiliate imports an intermediate input from the home country due to technological appropriability issues. Therefore, geographical costs apply to both exports and multinational production because they involve transportation of a finished good and an intermediate good, respectively. Upon drawing its own efficiency parameter, each firm decides whether to exit or to produce. In the latter case, firms must face additional fixed costs linked to the supply strategy chosen. If they serve the foreign market, they choose whether to export domestically produced goods or to produce abroad via affiliate production. Free entry ensures that firms continue to enter until the expected sum of future profits equals the cost of entry.

Our framework has features that yield new results for aggregate trade elasticity and welfare. Despite the free entry assumption, symmetry across countries allows us to retrieve margins of export and affiliate sales. The presence of two alternative ways of reaching the

foreign location introduces a double truncation in the productivity distribution of exporters. In particular, the upper bound on the productivity distribution, which is endogenously determined by the presence of multinational production, plays a crucial role in delivering an aggregate trade elasticity that is no longer constant, and depends on both supply and demand parameters.[1] The model is then calibrated to show that adding multinational production to export generates higher welfare gains by magnifying the response to trade openness.

This paper has three main contributions. Firstly, we derive gravity equations and margins' sensitivity for exports and affiliate sales. We find that, similarly to Chaney (2008), the intensive margin of export only depends on the elasticity of substitution. But, differently from Chaney (2008), we show that the extensive margin is not constant but is a function of both export and affiliate sales. We obtain a similar result for affiliate sales. More specifically, both margins are related to the elasticity of substitution and the share of the imported intermediate good. The extensive margin depends also on variable trade costs, and on the degree of firm heterogeneity.[2] Therefore, our model delivers elasticity measures, both for exports and affiliates, that depend on supply and demand parameters despite the Pareto assumption. This result relates to recent papers highlighting the limit of the Pareto and adopting alternative heavy-tailed productivity distributions, as Bas et al. (2017), Bee and Schiavo (2017), and Head et al. (2014).

Secondly, the standard results obtained for welfare in heterogeneous firm models with only exports are altered by the presence of multinational production. When the model includes multinational production, aggregate domestic trade shares and trade elasticity are no longer sufficient statistics to evaluate welfare gains. This result is in line with Feenstra (2014), Feenstra (2016) and Melitz and Redding (2015) which remove the long tail distribu-

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[1]Notice that intra-firm trade *per se* is not crucial for our findings. Also a model with pure multinational production, as in Helpman et al. (2004) would generate variable trade elasticity.

[2]That is, on the parameter that defines the shape of the productivity distribution.

tion related to the untruncted Pareto assumption, and introduce an exogenous upper bound on the productivity distribution. We develop a framework where the upper bound is *endogenously* determined by the presence of multinational firms. This alters the extensive margin elasticity of exports, which now depends on both export and multinational activities. As a consequence, welfare gains depend on foreign supply strategies, and not only on aggregate domestic trade shares and the standard constant trade elasticity. This result highlights the role of interdependent supply strategies for the aggregate trade elasticity and welfare.

Thirdly, we quantify the welfare gains from multinational production with intra-firm trade and examine the quantitative implications for both export and affiliate sales elasticities. We calibrate three versions of the model: (i) multinational production and intra-firm trade; (ii) multinational production *à la* Helpman et al. (2004); and, (iii) export only (with untruncated Pareto productivity distribution). Welfare gains from opening the closed economy to multinational production and intra-firm trade range from 5 to 20 percent. Trade liberalization can lead to welfare gains that are up to 4 percentage points higher than in models with untrucated Pareto distribution. Comparing welfare gains from our model to those in a model *à la* Helpman et al. (2004), we show that the latter has larger welfare gains from reduction in trade costs and smaller welfare losses from increase in trade costs. Welfare gains in a model *à la* Helpman et al. (2004) range from 12 to 20 percent. These are up to 10 percentage points higher than those generated by a model with untrucated Pareto distribution; and, up to 6 percentage points higher than those from our model with intra-firm trade. Finally, we compute the sensitivity of export and affiliate sales: our numbers for export elasticity are consistent with the estimation derived in the empirical literature (Mejean and Imbs, 2017 and Novy, 2013), and our affiliate sales elasticity ranges from 0.2 to 1.2 in absolute values.

Literature Review. This paper relates to several strands of research. It contributes to the

growing literature that theoretically analyzes the welfare gains from openness. Arkolakis et al. (2012) show that there exists a group of models in which a country's domestic trade share and the elasticity of trade are sufficient statistics to measure aggregate welfare gains from trade. This result relies on the assumption of an unbounded productivity distribution. Feenstra (2014), Feenstra (2016) and Melitz and Redding (2015) show that the additional adjustment margin in heterogeneous firm models plays an important role for welfare gains. Differently from these works, our welfare measure is altered by the endogenous double truncation in the productivity distribution of exporters, which responds to trade liberalization.

Another related strand of literature quantifies the gains from international activities. Edmond et al. (2015) study gains from international trade in a quantitative model with endogenously variable markups. Ramondo (2014) uses a multi-country general equilibrium model with a continuum of goods produced under constant return to scale at the industry level to calculate the gains that a country would experience from liberalizing access to foreign firms. Ramondo and Rodríguez-Clare (2013) introduce trade and multinational production into the Eaton-Kortum framework to measure the overall gains from openness. Garetto (2013) quantifies the gains from multinational activity, using an Eaton-Kortum type model, where multinational firms engage in vertical FDI. Irarrazabal et al. (2013) structurally estimate a model of trade and multinational production without free entry. They reject the proximity versus concentration hypothesis which did not consider intra-firm trade and find that impeding multinational activity has a small effect on welfare. Along the lines of these studies, we quantify the implications for trade elasticity and welfare of adding multinational production in a model with heterogeneous firms.

Lastly, the paper relates to the literature measuring aggregate trade elasticity. Models of heterogeneous firms with selection into export with an untruncated Pareto distribution

for productivity exhibit a constant trade elasticity. By contrast, Helpman et al. (2008) and Melitz and Redding (2015) show that for the bounded version of the Pareto, the trade elasticity recovers a bilateral-specific dimension. A recent strand of the literature investigates the limitation of the Pareto assumption, which represents a good approximation only for the right tail of the observed distribution of firm sizes. Head et al. (2014) and Bas et al. (2017) replace it with a lognormal distribution of productivity to obtain a better fit and a destination specific aggregate trade elasticity. We do not solve the empirical limitation of the Pareto distribution, but we propose a setup where the Pareto assumption maintains the analytical tractability and the coexistence of exporters and multinational firms allows the aggregate trade elasticity to recover the bilateral-specific dimension.

The rest of the paper is organized as follows. Section 2 describes the theoretical framework. Section 3 reports general equilibrium results. Section 4 discusses gravity equations, and intensive and extensive margins of export and affiliate sales. In Sections 5 and 6, we derive the theoretical implications of the model for welfare and provide comparative statics to highlight the additional channel. Section 7 contains the calibration. Finally, Section 8 concludes.

2. THEORETICAL FRAMEWORK

We use a model of export and horizontal multinational production with intra-firm trade to examine aggregate welfare implications. The model is solved assuming two perfectly symmetric countries and free entry. We assume two-tier preferences with Cobb-Douglas in the upper tier and CES in the lower tier. A consumer spends a fraction β of her income on $c(v)$ units of each variety v of the differentiated good, and $(1-\beta)$ on the homogeneous good h.

The utility function is

$$U = h^{(1-\beta)} \left[\int_{v \in V} c(v)^{(\sigma-1)/\sigma} dv \right]^{\frac{\sigma\beta}{\sigma-1}} \tag{2.1}$$

where $\sigma > 1$ represents the elasticity of substitution between any two products within the group and V is the set of available varieties. The homogeneous good h is freely traded and is used as the numeraire. It is produced under constant returns to scale with one unit of labor producing one units of good h. Its price is set equal to 1 so that the wage is 1. The differentiated sector produces a continuum of horizontally differentiated varieties, $x(v)$, from two intermediate goods, y_1 and y_2. Both y_1 and y_2 are produced with one unit of labor, but y_1 can only be made at home, due to technological appropriability issues. This assumption plays a crucial role for multinational production strategy: y_1 can be considered as transfer of capabilities between the headquarter and the foreign affiliate. Each variety is then supplied by a Dixit-Stiglitz monopolistically competitive firm which produces under increasing returns to scale originating from a fixed cost. We assume that the fixed cost is paid in units of labor.

Following Bombarda (2007) and Irarrazabal et al. (2013), we assume that the production of the final good is

$$x(v) = \frac{1}{a(v)} \left(\frac{y_1}{\eta} \right)^{\eta} \left(\frac{y_2}{1-\eta} \right)^{1-\eta} \qquad 0 < \eta < 1, \tag{2.2}$$

where $1/a(v)$ is the firm-specific productivity parameter and η is the Cobb-Douglas cost share of y_1. Using the intermediate results from the consumer and firm optimization problems, the operating profit from producing domestically is

$$\pi_d^*(a, A, \eta) = A a^{1-\sigma} \frac{1}{\sigma} \left(\frac{\sigma}{\sigma-1} \right)^{(1-\sigma)} - f_d, \quad A \equiv \frac{\beta E}{P^{1-\sigma}}$$

where A is the demand shifter, f_d is the domestic fixed cost, E is the aggregate level of spend-

ing, and P is the price index defined in Appendix A.1. Letting $B = \frac{A}{\sigma} \left(\frac{\sigma}{\sigma-1} \right)^{(1-\sigma)}$ we obtain

$$\pi_d^*(a, A, \eta) = Ba^{1-\sigma} - f_d. \tag{2.3}$$

If the firm chooses to reach the foreign market, it bears fixed costs f_x or f_m and variable cost τ, and its equilibrium net operating profit on sales in that market is

$$\pi_x^*(a, A, \eta) = B(\tau a)^{1-\sigma} - f_x, \text{ or} \tag{2.4}$$

$$\pi_m^*(a, A, \eta) = Ba^{1-\sigma}\tau^{\eta(1-\sigma)} - f_m, \tag{2.5}$$

depending on the mode of supply chosen, export or multinational production, respectively. We set parameters so as to obtain the same ranking as in Helpman et al. (2004). Namely, firms with sufficiently high productivity will supply the foreign market, with the most productive supplying it via multinational production rather than exports. In this way our model is in line with the empirical findings in Helpman et al. (2004). The regularity condition is

$$f_d < f_x \tau^{(\sigma-1)} < f_m \tau^{\eta(\sigma-1)}.$$

Using equations (2.3) to (2.5), the equilibrium cutoff conditions write as

$$a_d = \left(\frac{f_d}{B} \right)^{\frac{1}{1-\sigma}}, \tag{2.6}$$

$$a_x = \left(\frac{f_x}{B\tau^{1-\sigma}} \right)^{\frac{1}{1-\sigma}}, \tag{2.7}$$

$$a_m = \left(\frac{f_m - f_x}{B \left[\tau^{\eta(1-\sigma)} - \tau^{1-\sigma} \right]} \right)^{\frac{1}{1-\sigma}}. \tag{2.8}$$

3. GENERAL EQUILIBRIUM

This section presents general equilibrium results. Substituting the price index, which has been solved assuming the Pareto distribution (equation A.4), into the domestic cutoff condition (2.6), we obtain the equilibrium number of varieties (and hence the number of active firms) consumed in a typical country which depends on trade frictions related to different trade activities:

$$n^* = \frac{(b-1)\beta E}{\sigma b f_d \left[1 + T^{1-b}\phi^b + V^{1-b}\left(\phi^\eta - \phi\right)^b\right]},$$
(3.1)

where $b = \frac{k}{\sigma - 1}$; $\phi = \tau^{1-\sigma}$; $T = f_x/f_d$ and $V = (f_m - f_x)/f_d$.[3] k is the shape parameter of the productivity distribution described in Appendix A.2.

Using the price index and free entry condition in equations (A.4) and (A.6) , we can solve for the equilibrium domestic cutoff

$$a_d^* = a_0 \left[\frac{(b-1)f_e}{f_d(1 + \Psi + \Sigma)}\right]^{\frac{1}{k}},$$
(3.2)

where $\Psi = V^{1-b}\left[\phi^\eta - \phi\right]^b$ and $\Sigma = T^{1-b}\phi^b$. Replacing (3.2) into the ratio between (2.7) and (2.6), we find

$$a_x^* = a_0 \left[\frac{(b-1)f_e}{f_x(1 + \Psi + \Sigma)}\phi^b T^{1-b}\right]^{\frac{1}{k}}.$$
(3.3)

Adopting a similar strategy, we obtain the equilibrium cutoff for the M-mode

$$a_m^* = a_0 \left[\frac{(b-1)f_e}{(f_m - f_x)(1 + \Psi + \Sigma)}\left[\phi^\eta - \phi\right]^b V^{1-b}\right]^{\frac{1}{k}}.$$
(3.4)

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[3]Appendix A provides further details on the price index and free entry solved under the Pareto distribution (equations (A.4) and (A.6), respectively).

4. INTENSIVE AND EXTENSIVE MARGINS

In this section we derive intensive and extensive margins of export and affiliate sales.

4.1. Export Sales

We differentiate the expression of total exports of a country $X^x = n \int_{a_m}^{a_x} x^x \mathrm{d}G(a)$ with respect to variable trade costs and derive the intensive and extensive margins of export sales

$$\frac{\partial X^x}{\partial \tau} = \underbrace{n \int_{a_m}^{a_x} \frac{\partial x^x}{\partial \tau} \mathrm{d}G(a)}_{\text{Intensive Margin}} + \underbrace{n \left[x^x G'(a_x) \frac{\partial a_x}{\partial \tau} - x^m G'(a_m) \frac{\partial a_m}{\partial \tau} \right]}_{\text{Extensive Margin}}, \qquad (4.1)$$

where we applied the Leibniz rule to separate the margins, and assume that the number of active firms is constant.[4]

PROPOSITION 1. *Elasticity of export sales is no longer constant. Let $\Omega \equiv -\partial \log X^x / \partial \log \tau$, a change in the variable costs τ makes the margins of export sales to react as follows*

$$\Omega = \underbrace{(\sigma - 1)}_{\text{Intensive Margin Elasticity}} + \underbrace{(k - \sigma + 1)\left[1 - \frac{X^m}{X^x}(\Gamma - \omega) \right]}_{\text{Extensive Margin Elasticity}}, \qquad (4.2)$$

where

$$\Gamma = \frac{\eta \tau^{(1-\sigma)\eta} - \tau^{(1-\sigma)}}{b(\tau^{(1-\sigma)\eta} - \tau^{(1-\sigma)})}, \qquad (4.3)$$

$$\omega = \tau^{(1-\eta)(1-\sigma)}. \qquad (4.4)$$

Proof. See Appendix D. $\qquad \qquad \qquad \square$

Intensive Margin. Similarly to models with untruncated Pareto distribution, the volume of

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[4]The denominator in (3.1) does not change. This is similar to the assumption on θ made by Chaney (2008).

export sales depends on the constant elasticity of substitution. This implies that when goods are very substitutable (high σ), sales of each exporter are very sensitive to the trade barriers.

Extensive Margin. *Differently from models with an untruncated Pareto, the extensive margin in our model depends on variable trade costs and it is not constant at* $(k - \sigma + 1)$. Equation (4.2) shows that the sensitivity of the extensive margin of exports to trade policy depends on the interaction between aggregate affiliate and export sales.[5] This is because the change in the number of varieties supplied via exports depends on the level of profits generated by export and multinational production strategies, which in turn affect overall affiliate sales.

Let us focus on the second part of (4.2). If $X^m > X^x$, a decrease in trade cost reduces the extensive margin elasticity of export. Notice that the sign of the overall elasticity depends on the size of the intensive margin, which can compensate the negative extensive margin. When $X^m < X^x$ the opposite is true, and a decrease in trade costs increases the extensive margin. To summarize, while the elasticity of the intensive margin is always positive (a decrease in trade costs increases the volume of trade), the behaviour of the extensive margin depends on how export and affiliate sales interact.

Differently from Chaney (2008), the elasticity of exports with respect to variable costs depends on the elasticity of substitution between goods, σ, and on trade costs τ. The endogenously bounded productivity framework reaffirms the importance of trade costs and elasticity of substitution in models of firm heterogeneity, in line with the results of Head et al. (2014) and Bas et al. (2017).

4.2. Affiliate Sales

In this section we analyze how elasticity of substitution and the share of intermediate inputs affect the sensitivity of affiliate sales' margins. Differentiating total affiliate sales of

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[5]Note that $\Gamma > \omega$ is true for certain parameter restrictions consistent with our calibration. For further details on Γ and ω, see Appendix D.

a country $X^m = n \int_0^{a_m} x^m \mathrm{d}G(a)$ with respect to variable trade cost, we derive the intensive and extensive margins of affiliate sales

$$\frac{\partial X^m}{\partial \tau} = \underbrace{n \int_0^{a_m} \frac{\partial x^m}{\partial \tau} \mathrm{d}G(a)}_{\text{Intensive Margin}} + \underbrace{n x^m G'(a_m) \frac{\partial a_m}{\partial \tau}}_{\text{Extensive Margin}}, \tag{4.5}$$

where we applied the Leibniz rule to separate the margins.

PROPOSITION 2. *Elasticity of affiliate sales is no longer constant. Let $\psi \equiv -\partial \log X^m / \partial \log \tau$, a change in variable costs τ makes the margins of affiliate sales react as follows*

$$\psi = \underbrace{\eta(\sigma-1)}_{\text{Intensive Margin Elasticity}} + \underbrace{\frac{(k-\sigma+1)}{k} \frac{(\sigma-1)(\eta \tau^{(1-\sigma)\eta} - \tau^{1-\sigma})}{\tau^{(1-\sigma)\eta} - \tau^{(1-\sigma)}}}_{\text{Extensive Margin Elasticity}}, \tag{4.6}$$

Proof. See Appendix E.   □

Intensive Margin. The intensive margin of affiliate sales depends on the constant elasticity of substitution and on the level of imported intermediate η. Therefore, when goods are very substitutable (high σ), sales of each individual affiliate are very sensitive to the trade barriers. Let us now focus on the role of the parameter η.

When η is equal to one, no firm will supply via multinational production. In this case, the foreign affiliate is importing both intermediate inputs from the home country. This strategy is extremely costly, since it implies bearing full trade costs as well as higher fixed costs. Therefore, export is the only market access strategy. Differently, when η is equal to zero, the foreign affiliate is producing using only foreign inputs (similarly to Helpman et al., 2004). When all intermediates are realized in the foreign location, the volume of sales of already existing affiliates are not affected by changes in trade costs. Therefore, the intensive margin elasticity is equal to zero.

For intermediate levels of η, both the extensive and the intensive margins of affiliate sales are affected by the intensity of imported headquarter intermediates. The behavior of the intensive margin is unambiguous, and σ magnifies its sensitivity. When σ is high, the change in X^m due to a change in τ is mostly captured by the intensive margin: if τ decreases, new affiliates enter the market, but a high σ leads to a high level of competition. In this environment, having a low productivity is an even bigger disadvantage as firms can only capture a small market share, and their impact on the overall affiliate sales is small.

Extensive Margin. The sensitivity of the extensive margin of affiliate sales to changes in trade costs is not constant and it is related to τ, σ, k, and η. In general, we should expect that when the substitutability across varieties is low, an increase in σ makes entrance of new affiliates more sensitive to changes in τ. On the one hand, trade liberalization makes it easier to import the intermediate goods; on the other hand, the low degree of substitution keeps the level of competition down. This explains why more firms can survive as new affiliates after entry. Contrarily, a larger degree of substitutability among varieties makes entry of new affiliates less sensitive to changes in τ. In fact, when the level of competition is high, new entrants will capture only a small fraction of market share despite the reduction in trade costs.

To further stress this result, in the calibration section we proposes an exercise to understand the effects of trade policy on both export and affiliate sales' margins.

5. WELFARE GAINS WITH EXPORTS AND INTRA-FIRM TRADE

In this section we derive the aggregate trade elasticity and isolate the additional component of welfare associated with multinational firms. This exercise shows that welfare gains in heterogeneous firms model with export and multinational are altered.

Aggregate trade between two countries is inversely related to the domestic trade share, which can be written as the proportion of domestic sales in total sales, that is

$$\lambda \;=\; \frac{\int_0^{a_d} r_d \, \mathrm{d}G(a/a_d)}{\int_0^{a_d} r_d \, \mathrm{d}G(a/a_d) + \int_{a_m}^{a_x} r_x \, \mathrm{d}G(a/a_d) + \int_0^{a_m} r_m \, \mathrm{d}G(a/a_d)}. \tag{5.1}$$

Setting $\Lambda = \frac{\int_{a_m}^{a_x} a^{1-\sigma} \, dG(a/a_d)}{\int_0^{a_d} a^{1-\sigma} \, dG(a/a_d)}$ and $Z = \frac{\int_0^{a_m} a^{1-\sigma} \, dG(a/a_d)}{\int_0^{a_d} a^{1-\sigma} \, dG(a/a_d)}$, the domestic trade share becomes

$$\lambda = \frac{1}{1 + \tau^{1-\sigma} \Lambda + \tau^{\eta(1-\sigma)} Z}, \tag{5.2}$$

where the term $\tau^{\eta(1-\sigma)} Z$ is related to multinational production.

As argued by Arkolakis et al. (2012), only the partial trade elasticity capturing the effect of τ is observed empirically. To derive the partial trade elasticity in our model we use (5.2), which relates the domestic trade share to variable trade costs and the three cutoffs ($\lambda = \lambda(\tau, a_d, a_x, a_m)$), and from (2.7) and (2.8). Taking the partial derivative of the domestic trade share with respect to τ holding a_d constant, we obtain:

$$
\begin{aligned}
\Xi = \frac{\partial \ln \lambda}{\partial \ln \tau} \;&=\; \frac{\partial \lambda}{\partial \tau} \frac{\tau}{\lambda} \\
&=\; -\frac{1}{\lambda} \left\{ (1-\sigma)\left[\tau^{1-\sigma} \Lambda + \eta \tau^{\eta(1-\sigma)} Z \right] + \tau\left[\tau^{1-\sigma} \frac{\partial \Lambda}{\partial \tau} + \tau^{\eta(1-\sigma)} \frac{\partial Z}{\partial \tau} \right] \right\} \\
&=\; \frac{(\sigma-1)}{\lambda} \left\{ \left[\tau^{1-\sigma} \Lambda + \eta \tau^{\eta(1-\sigma)} Z \right] - \left[\gamma(\varphi_x) \frac{f_x}{B} + \gamma(\varphi_m) \frac{(f_m - f_x)}{B} \tilde{\tau} \right] \right\} \quad (5.3)
\end{aligned}
$$

where $\varphi_j = a_j^{1-\sigma}$ for $j \in \{d, x, m\}$, $\gamma(\varphi_x) = \partial \Lambda / \partial \varphi_x$, $\gamma(\varphi_m) = \partial Z / \partial \varphi_m$, and $\tilde{\tau} = (\eta \tau^{(1-\sigma)(2+\eta)} - \tau^{(1-\sigma)(1+\eta)})/(\tau^{\eta(1-\sigma)} - \tau^{1-\sigma})^2$. It is important to notice that the partial elasticity in (5.3) is not constant (and equal to k), but it depends on the response of mode of supply to trade costs. As a consequence, welfare gains in our model are different from welfare gains in Arkolakis et al. (2012) and in Melitz and Redding (2015).

To retrieve an expression for welfare gains, we divide the price index in equation (A.3) by $\int_0^{a_d} a^{1-\sigma} dG(a/a_d)$. Then solving for P we can express welfare as a function of the domestic trade share, λ,

$$\frac{w}{P} = \frac{\sigma-1}{\sigma} n^{\frac{1}{\sigma-1}} \left(\frac{\delta(\varphi_d)}{\lambda} \right)^{\frac{1}{\sigma-1}}, \tag{5.4}$$

where we set $\delta(\varphi_d) = \int_0^{a_d} a^{1-\sigma} \, dG(a/a_d)$. Welfare gains can be derived by log-differentiating equation (5.4), which gives:

$$
\begin{aligned}
\mathrm{d}\ln \frac{w}{P} &= \frac{1}{\sigma-1} \left[\mathrm{d}\ln n + \mathrm{d}\ln \delta(\varphi_d) - \mathrm{d}\ln \lambda \right] \\
&= \frac{\vartheta}{\Xi \lambda} \left[\mathrm{d}\ln n + \mathrm{d}\ln \delta(\varphi_d) - \mathrm{d}\ln \lambda \right]
\end{aligned}
\tag{5.5}
$$

where we used the partial trade elasticity from (5.3) setting $\vartheta = \left\{ \left[\tau^{1-\sigma}\Lambda + \eta\tau^{\eta(1-\sigma)}Z \right] - \left[\gamma(\varphi_x)\frac{f_x}{B} + \gamma(\varphi_m)\frac{(f_m-f_x)}{B}\tilde{\tau} \right] \right\}$.

Equation (5.5) shows that welfare gains from trade liberalization are affected by the micro structure, which change with trade liberalization. Specifically, welfare gains reflect the interaction between exports and multinational activities (ϑ) and domestic cutoff ($\mathrm{d}\ln\delta(\varphi_d)$), other than domestic trade share and trade elasticity. Therefore, changes in the micro structure related to the mode of supply will affect welfare gains from trade. This result complements the findings of Melitz and Redding (2015) by showing the role of interdependent supply strategies for the partial trade elasticity. In our model the partial trade elasticity is variable and the nature of this variability rests on the different mode of supply. In the next section we show that adding multinational firms reinforces the welfare gains from trade liberalization in model with firm heterogeneity.

6. THEORETICAL COMPARATIVE STATICS

In this section we compare the aggregate welfare properties of the model described in section 2 (henceforth, IF-model) to a model with only export and no upper bound (henceforth, EX-model), and then to a model of export and multinational production *à la* Helpman et al. (2004) (henceforth, HMY-model). Using the price index in equation (A.4), we can obtain a measure of welfare for our model with multinational production and intra-firm trade:

$$
\begin{aligned}
W_{IF} &= \frac{w}{P} \\
&= \frac{\sigma}{\sigma-1}\left(\frac{1}{1-1/b}\right)^{\frac{1}{\sigma-1}}\left[\frac{(b-1)\beta E}{\sigma b f_d}\right]^{\frac{1}{\sigma-1}}\left[\frac{f_d}{(b-1)f_e}\right]^{\frac{1}{k}}\left(1+T^{1-b}\phi^b+V^{1-b}\left(\phi^\eta-\phi\right)^b\right)^{\frac{1}{k}},
\end{aligned}
\tag{6.1}
$$

where we used the equilibrium values for n^* and a_d from equations (3.1) and (3.2), respectively. By shutting down the multinational production channel, we can retrieve the corresponding welfare measure for a model with export, which becomes:

$$
W_{EX} = \frac{w}{P} = \frac{\sigma}{\sigma-1}\left(\frac{1}{1-1/b}\right)^{\frac{1}{\sigma-1}}\left[\frac{(b-1)\beta E}{\sigma b f_d}\right]^{\frac{1}{\sigma-1}}\left[\frac{f_d}{(b-1)f_e}\right]^{\frac{1}{k}}\left(1+T^{1-b}\phi^b\right)^{\frac{1}{k}}.
\tag{6.2}
$$

Welfare in a model *à la* Helpman et al. (2004) can be obtained by setting $\eta = 0$ in equation (6.1).

PROPOSITION 3. *Given the same value for the parameters $\{f_d, f_e, f_x, k, L, \sigma\}$, welfare in the IF-model is higher than welfare in the EX-model. Welfare in the HMY-model is the highest: $W_{HMY} > W_{IF} > W_{EX}$.*

Proof. Comparison between equations (6.1) and (6.2) immediately shows that $W_{IF} > W_{EX}$ since $1+T^{1-b}\phi^b+V^{1-b}\left(\phi^\eta-\phi\right)^b > 1+T^{1-b}\phi^b$. When $\eta = 0$, then $V^{1-b}\left(1-\phi\right)^b > V^{1-b}\left(\phi^\eta-\phi\right)^b$, which implies that welfare in the HMY-model is the highest. Thus, $W_{HMY} > W_{IF} > W_{EX}$. □

Next, consider the effect of a reduction in trade costs.

PROPOSITION 4. *Departing from the same level of welfare, the proportional welfare gains from reducing trade costs are strictly larger in a model where multinational production exists. The largest gains are obtained in the HMY-model.*

Proof. The proof follows from Proposition 3. In particular we need to show that welfare in the IF-model is higher than in the EX-model for any level of trade costs. This is straightforward from comparing equations (6.1) to (6.2). □

Therefore, in a model with exports and intra-firm trade, decreasing trade costs generates larger welfare gains than in a model with only exports. This happens because trade costs affect both types of activities, export and intra-firm trade. For the same reason, welfare losses from an increase in trade costs are smaller in a world with intra-firm trade. The existence of different types of firms engaged in international trade, which have different degrees of exposure to trade barriers, increases the advantages of trade liberalization.[6]

7. QUANTITATIVE EXERCISE

We examine the quantitative relevance of our model. First, we show that there are economically relevant differences in welfare, mass of active firms, and domestic trade share between the model with only export and no upper bound (EX-model) and our model with multinational production and intra-firm trade (IF-model). Then, in section 7.1 we examine welfare gains from a model *à la* Helpman et al. (2004) (HMY-model). Finally, in section 7.2, we analyze the sensitivity of intensive and extensive margins of exports and affiliate sales to variable trade costs.

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[6]This results is stronger when η is sufficiently small. Notice that when $\eta \longrightarrow 0$ we are in Helpman et al. (2004).

We set the elasticity of substitution $\sigma = 4.25$, as in Broda and Weinstein (2006): over the 1999-2001 period they find average and median elasticities for SITC 5-digit goods of 13.1 and 2.7, respectively (see their Table IV).[7] Consistently with the literature, we choose the shape parameter of the Pareto distribution to be $k = 4.25$.[8] Geographical and trade barriers are set to $\tau = 1.83$, as in Melitz and Redding (2015) and Irarrazabal et al. (2013). We consider trade between two symmetric countries, and choose labor in one country as the numeraire ($w = 1$), which implies that the wage in both countries is equal to one. We set L equal to the U.S. labor force; we normalize f_d and f_e to one and set $f_x = 0.545$ as in Melitz and Redding (2015).

Given our choice for the parameters $\{\sigma, k, f_d, f_e, f_x, \tau\}$, we choose f_m to ensure that the model generates the average fraction of U.S. manufacturing firms that export of 18 percent (Bernard et al., 2007), and η to match the volume of intra-firm sales as a percentage of total export (20.1 percent, as in Table 11 of Mataloni and Yorgason, 2006). In our baseline specification we compute the open economy equilibrium using our calibrated values of $f_m = 2.85$, $\eta = 0.33$, and trade costs of $\tau = 1.83$.

In Figure 1, we show the effects of adjusting variable trade costs from their calibrated value of $\tau = 1.83$ (vertical line) to $\tau \in [1, 2]$. For the sake of comparison, we also calibrate an economy where we shut-down intra-firm trade and multinational production, the percentage of exporting firms is 18 percent, and find $f_x = 0.545$ as in Melitz and Redding (2015). Panel A displays the welfare gains from opening the closed economy to multinational production and intra-firm trade. As shown in Proposition 3, welfare gains from opening an economy to export are strictly higher when there is multinational production. Our numerical example delivers welfare gains from the IF-model that are up to four 4 percentage points

---

[7]The value $\sigma = 4.25$ implies a mark-up of 31 percent.

[8]Bernard et al. (2003), Eaton et al. (2011), and more recently Simonovska and Waugh (2014) found estimates of the shape parameter from firm-level sales data in the range of 3.6 to 4.8.

higher the those from the EX-model. In Panel B we plot welfare gains with multinational production relative to welfare gains with only exports. Welfare gains in both economies are decreasing in τ (Panel A), but they decrease faster in an economy with only exports.
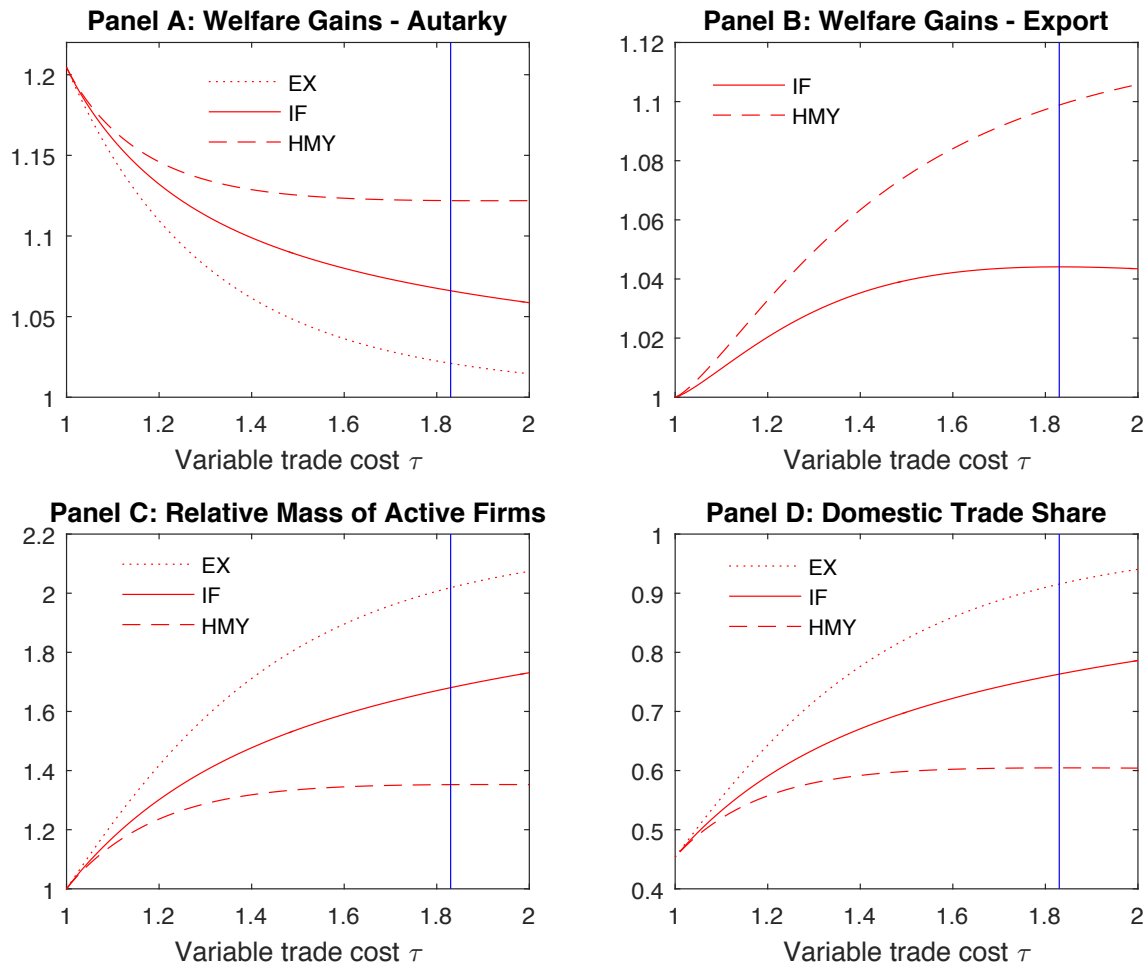

Figure 1 – Variable trade costs

Panel C shows the relative (to $\tau = 1$) mass of active firms. An increase in variable trade costs raises the number of actives in both economies relative to the mass of actives for $\tau = 1$. But this number is strictly lower in a model with multinational production due to the higher domestic cutoff productivity. The number of active firms increases faster with τ when only exporting firms are considered. Panel D displays the domestic trade share λ, which is higher for an economy with only exporting firms. It increases faster with τ in the EX-model because, when also multinational firms exist, the denominator of equation (5.1) is larger.

In Figure 2, we show the effects of adjusting the share of intra-firm trade from their calibrated value of 0.33 to $\eta \in [0, 1]$.
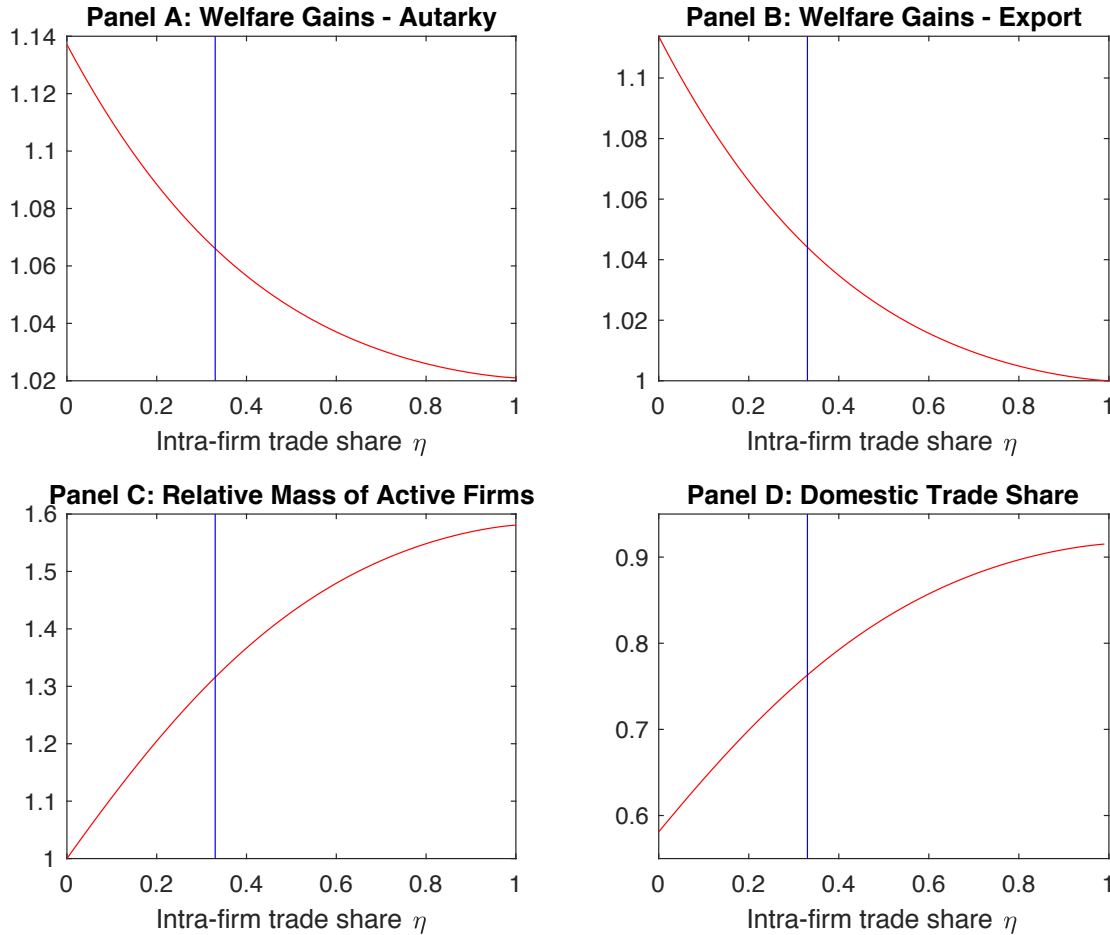

Figure 2 – Increasing share of intra-firm trade

An increase in η acts similarly to an increase in variable trade costs which exclusively affects multinational firms, i.e. the most productive firms. Similarly to an increase in τ, the increase in η reduces average productivity by diminishing the level of competition in each market. This implies a larger amount of active firms in equilibrium (Panel C). Panel A shows that an increase in η reduces welfare gains because of the raise in the proportion of less productive firms. Panel B shows that welfare gains from allowing exporting firms to produce in foreign countries are still decreasing in η. The insights are similar to these obtained from Panel A, as the denominator (welfare gains from export) does not depend on η. Panel D plots

λ as an increasing function of η. As η goes towards 1, it becomes more and more difficult to do intra-firm trade, and therefore to undertake multinational production with the result of increasing domestic trade share.

7.1. Multinational Production à la Helpman et al. (2004)

We now consider an economy where $\eta = 0$, and all production is carried out in the host country, i.e. the HMY-model. The percentage of exporting firms is calibrated to 18 percent, with $f_x = 0.545$ and $f_m = 4.485$. Figure 3 shows the corresponding welfare gains, active firms, and domestic trade share.
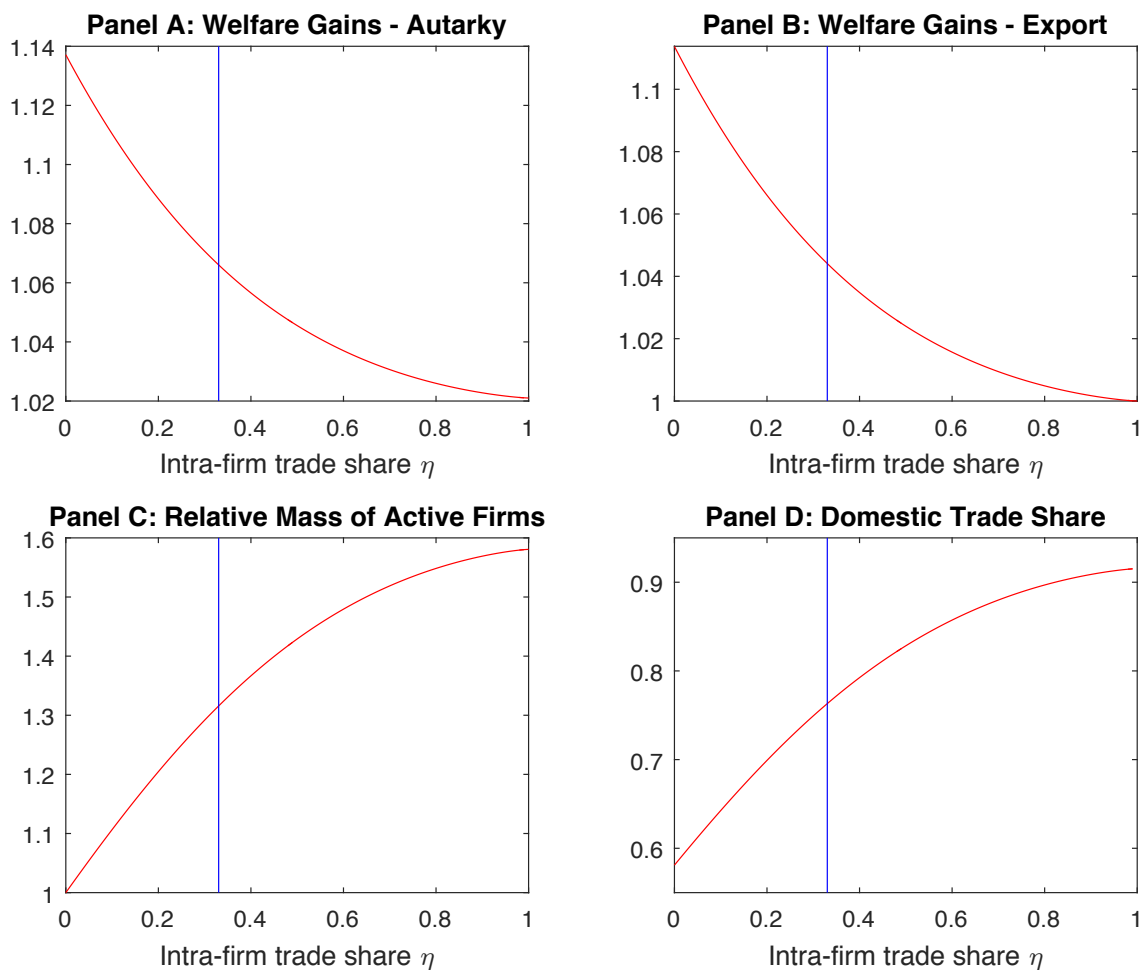

Figure 3 – Variable trade costs

Panel A shows that welfare gains from the HMY-model are strictly higher than welfare

gains from the other models. In all economies, welfare gains increase as τ decreases but they are up to 10 percent higher in HMY-model than in the EX-model, and up to 6 percent higher than in the IF-model. This is related to Proposition 3, and it is due to the higher average domestic productivity required to enter the market and operate.

Panel B plots the relative (to the EX-model) welfare gains. The figure underlines that, in comparison with an exporting economy, welfare gains from the HMY-model decrease at a lower rate than they do with intra-firm trade. The reason is that only exporters face trade frictions. A higher average productivity in the HMY-model explains the lower relative mass of active firms (Panel C), and the lower domestic trade share (Panel D).

In Table 1 we show the results of another exercise in which welfare gains are computed varying σ and k. Welfare gains from multinational production firms are computed with respect to a closed economy (W_{Aut}), and to an exporting economy (W_{EX}). Welfare gains decrease when market power increases (lower σ), and the drop is higher when outsourcing is total, i.e. in the HMY-model. A decrease in k, the shape parameter of the Pareto distribution, increases the productivity cutoff of domestic firms and raises welfare gains.

Table 1 – Comparative Statics - Welfare Gains

	Baseline Calibration $\sigma = 4.25, k = 4.25$	$\sigma = 3.5$	$\sigma = 4.5$	$k = 3.75$	$k = 4.75$
	(1)	(2)	(3)	(4)	(5)
W_{IF}/W_{Aut}	1.066	1.049	1.072	1.095	1.047
W_{IF}/W_{EX}	1.044	1.022	1.051	1.064	1.031
W_{HMY}/W_{Aut}	1.122	1.077	1.137	1.169	1.089
W_{HMY}/W_{EX}	1.099	1.049	1.115	1.135	1.073

7.2. Elasticities of Export and Affiliate Sales

Using equations (4.2) and (4.6), we now examine the quantitative implications for export and affiliate sales elasticities. In Figure 4 we show the elasticity of export sales (Panel A), and affiliate sales (Panel B).[9]
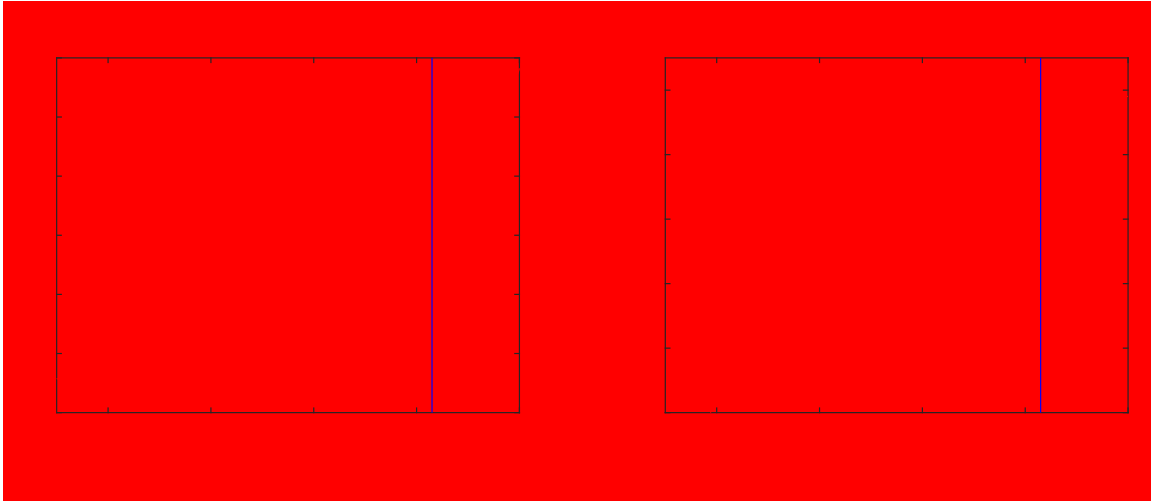

Figure 4 – Trade Elasticity

As we vary variable trade costs from one to two, export elasticity in Panel A ranges from less than seven to more than twelve. These numbers are in line with the findings of the empirical literature: Mejean and Imbs (2017) structural estimates' range from less than four to more than nine; and, Novy (2013) estimates' vary from less than five to more than twenty.

As variable trade costs increase, the export productivity cutoff rises at a higher rate than intra-firm trade productivity cutoff, which in turn increases the export elasticity.[10] Hence, we show that the presence of multinational firms has an impact on the extensive margin elasticity of export which varies across markets. Panel B shows that affiliate sales elasticity rises at a decreasing rate, in line with the intra-firm productivity cutoff.

---

[9]Figure 4 reports elasticities in absolute values.

[10]Equation (4.1) shows that the variable part of export elasticity is a function of the difference between productivity cutoffs.

8. CONCLUSION

The goal of this paper has been to analyze the implication for trade elasticity and welfare of using multinational production as an endogenous upper bound on the productivity distribution. An important theoretical result of the paper is that alternative market access strategies alter the standard results obtained for welfare in heterogeneous firm models, through a double truncated productivity distribution. In particular, the endogenous upper bound on the Pareto productivity distribution generates an aggregate trade elasticity that is no longer constant, but depends on supply and demand parameters despite the Pareto assumption. Therefore, our welfare gains are affected by the micro structure other than country's domestic trade share and trade elasticity.

To quantitatively assess the welfare gains from multinational production with intra-firm, we calibrated the model to match aggregate U.S. data. Trade liberalization can lead to welfare gains that are up to 4 percentage points higher than in models with untrucated Pareto distribution. Comparing welfare gains from our model to those in a model *à la* Helpman et al. (2004), we show that the latter has larger welfare gains from reduction in trade costs and smaller welfare losses from increase in trade costs.

ACKNOWLEDGMENTS

A. APPENDIX

A. Free Entry and the Price Index in the Symmetric Case

A.1 Free Entry

We describe the equilibrium which characterizes this perfectly symmetric economy. In order to do so, we need to specify some other equilibrium equations, namely the free-entry condition and the price index.

Free entry ensures equality between the expected operating profits of a potential entrant and the entry cost, $E(\pi) - f_e$. This condition holds for all types of firms. The cumulative density function is $G(a)$, with support: $[0, ..., a_0]$. The free-entry condition can be defined as

$$f_e = \int_0^{a_d} \pi_d dG(a) + 2\Big(\int_{a_m}^{a_x} \pi_x dG(a) + \int_0^{a_m} \pi_m dG(a) \Big) \tag{A.1}$$

Using the profit conditions (2.3)-(2.5), we obtain

$$
\begin{aligned}
f_e \;=\; & \int_0^{a_d} \Big[\Big(\frac{\sigma}{\sigma-1}\Big)^{(1-\sigma)} \frac{\beta E a^{1-\sigma}}{P^{1-\sigma}\sigma} - f_d\Big] \mathrm{d}G(a) + 2\int_{a_m}^{a_x} \Big[\Big(\frac{\sigma}{\sigma-1}\Big)^{(1-\sigma)} \frac{\phi\beta E a^{1-\sigma}}{P^{1-\sigma}\sigma} - f_x\Big] \mathrm{d}G(a) \\
& +2\int_0^{a_m} \Big[\Big(\frac{\sigma}{\sigma-1}\Big)^{(1-\sigma)} \frac{\phi^\eta \beta E a^{1-\sigma}}{P^{1-\sigma}\sigma} - f_m\Big] \mathrm{d}G(a),
\end{aligned}
\tag{A.2}
$$

where $\phi = \tau^{1-\sigma}$ is freeness of trade, and $P^{1-\sigma}$ is a weighted average of the marginal costs corrected for markups of all firms active in the market.

We now analyze in detail the term $P^{1-\sigma}$. This weighted average, $P^{1-\sigma}$, is characterized by all the varieties offered in each country: the varieties offered by domestic firms, for which the consumer price is $a\sigma/(\sigma-1)$; the varieties offered by foreign exporters, for which the consumer price is $a\sigma\tau/(\sigma-1)$; and, finally, the varieties supplied by foreign subsidiaries,

with consumer price $a\sigma (\tau)^{\eta} / (\sigma - 1)$. Therefore

$$
\begin{aligned}
P^{1-\sigma} &= \left(\frac{\sigma}{\sigma-1}\right)^{(1-\sigma)} n \int_0^{a_d} a^{1-\sigma} \mathrm{d}G(a/a_d) + \\
&\quad \left(\frac{\sigma}{\sigma-1}\right)^{(1-\sigma)} n \Big[\int_0^{a_m} \phi^{\eta} a^{1-\sigma} \mathrm{d}G(a/a_d) + \int_{a_m}^{a_x} \phi a^{1-\sigma} \mathrm{d}G(a/a_d) \Big]
\end{aligned} \tag{A.3}
$$

where n is the measure of varieties available in the country.

A.2   Parameterization: Pareto Distribution

The fact that the free-entry condition and the price index depend on the probability distribution implies that, in order to have explicit solutions for this model, we need to assume a particular functional form for $G(a)$. Following the empirical literature on firm-size distributions (see Axtell, 2001 and Helpman et al., 2004), we use the Pareto distribution as an approximation. The cumulative distribution function of a Pareto random variable a is $G(a) = (a/a_0)^k$, where k and a_0 are the shape and scale parameters, respectively. Note that $k = 1$ implies a uniform distribution on $[0, a_0]$. The shape parameter k represents the dispersion of cost draws. An increase in k would imply less dispersion in the firm productivity draws: the higher is k the less heterogeneity there is. We can now use this Pareto distribution to derive the price index and the free entry condition.

As noted above, firms offer a price only if they have productivity of at least $1/a_d$. The cumulative distribution is hence defined over a support $[0, ..., a_d]$. We can now solve the symmetric price index to obtain

$$
P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{(1-\sigma)} \frac{n}{1-\frac{1}{b}} a_d^{1-\sigma} \left[1 + T^{1-b} (\phi)^b + V^{1-b} \left[(\phi)^{\eta} - \phi \right]^b \right], \tag{A.4}
$$

where $b = \frac{k}{\sigma-1}$; $T = f_x/f_d$ and $V = (f_m - f_x)/f_d$. In order for the integral to converge we as-

sume that $b > 1$. Rewriting now the free entry condition in (A.1) using the Pareto distribution we obtain

$$\left(\frac{\sigma}{\sigma-1}\right)^{(1-\sigma)} \frac{\beta E}{\sigma P^{1-\sigma}} \Big[\int_0^{a_d} a^{1-\sigma} \mathrm{d}G(a) + \int_0^{a_m} (\phi)^\eta a^{1-\sigma} \mathrm{d}G(a) + \int_{a_m}^{a_x} \phi a^{1-\sigma} \mathrm{d}G(a) \Big] \tag{A.5}$$
$$= f_d G(a_d) + \big(f_x G(a_x) - f_x G(a_m) + f_m G(a_m) \big) + f_e.$$

B. Aggregate Sales with Free Entry

Let's define the aggregate affiliate sales as

$$\begin{aligned} X^m &= n^* \int_0^{a_m} a^{1-\sigma} A \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} (\tau)^{(1-\sigma)\eta} \, dG(a/a_d) \\ &= n^* \tau^{(1-\sigma)\eta} \left(\frac{a_m}{a_d}\right)^k a_m^{1-\sigma} \frac{k}{k-\sigma+1} A \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}, \end{aligned} \tag{A.6}$$

where $A \equiv \frac{\beta E}{P^{1-\sigma}}$. Similarly, aggregate export sales are

$$\begin{aligned} X^x &= n^* \int_{a_x}^{a_m} \tau^{1-\sigma} a^{1-\sigma} A \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} dG(a/a_d) \\ &= n^* \tau^{1-\sigma} A \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \frac{k}{a_d^k} \frac{\left(a_x^{k-\sigma+1} - a_m^{k-\sigma+1}\right)}{k-\sigma+1}. \end{aligned} \tag{A.7}$$

Since in the above expressions we are conditioning on a_d, to find aggregate sales we simply multiplied by number of active n^*.

C. Number of MNF and Exporting Firms

The number of affiliates producing in each country is given by

$$n_m = n^* \left(\frac{a_m}{a_d}\right)^k, \tag{A.8}$$

where n^* is the number of active firms and comes from equation (3.1). The number of exporting firms in each country is obtained from

$$n_x = n^* \left(\frac{a_x}{a_d} \right)^k. \qquad \text{(A.9)}$$

D. Intensive and Extensive Margins of Export Sales

1) Rearranging the definition of intensive and extensive margins of exports we get

$$-\frac{\partial X^x}{\partial \tau} \frac{\tau}{X^x} = \underbrace{-\frac{\tau}{X^x} \left(n \int_{a^m}^{a} \frac{\partial x^x}{\partial \tau} dG(a) \right)}_{\text{Intensive Margin Elasticity}} \underbrace{-\frac{\tau}{X^x} n \left[x^x G'(a) \frac{\partial a}{\partial \tau} - x^m G'(a^m) \frac{\partial a^m}{\partial \tau} \right]}_{\text{Extensive Margin Elasticity}}.$$

$$\text{(A.10)}$$

Using the definition of equilibrium individual export sales, which is

$$x^x = p^x q^x = \frac{\sigma}{\sigma - 1} a\tau \beta E \frac{(a\tau \frac{\sigma}{\sigma-1})^{-\sigma}}{P^{1-\sigma}} = \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} A(a\tau)^{1-\sigma}, \qquad \text{(A.11)}$$

and following the literature we consider only the partial elasticity and get

$$
\begin{aligned}
\frac{\partial x^x}{\partial \tau} &= (1-\sigma) \tau^{-\sigma} a^{1-\sigma} A \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \\
&= (1-\sigma) \frac{x^x}{\tau}. \qquad \text{(A.12)}
\end{aligned}
$$

Therefore, the elasticity of the intensive margin of export with respect to the variable costs is:

$$
\begin{aligned}
\varepsilon_{I,\tau}^x &= -\frac{\tau}{X^x} \left(n \int_{a^m}^{a} \frac{\partial x^x}{\partial \tau} dG(a) \right) \\
&= -(1-\sigma) \frac{\tau}{X^m} \frac{n \int_{a^m}^{a} x^x dG(a)}{\tau} \\
&= (\sigma - 1), \qquad \text{(A.13)}
\end{aligned}
$$

which is identical to the elasticity in Chaney (2008).

2) In order to derive the extensive margin of trade we need to use the equilibrium pro-
ductivity thresholds from (3.3) and (3.4). Deriving these thresholds with respect to τ
we find:

$$\frac{\partial a}{\partial \tau} = -\frac{a}{\tau}, \tag{A.14}$$

and

$$\frac{\partial a^m}{\partial \tau} = -\frac{a_m}{k}\frac{(\sigma-1)(\eta\tau^{(1-\sigma)\eta-1}-\tau^{(-\sigma)})}{\tau^{(1-\sigma)\eta}-\tau^{(1-\sigma)}}. \tag{A.15}$$

Rewriting the equation for firm level exports in (A.11), we obtain

$$\begin{aligned} x^x &= \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} A(a\tau)^{1-\sigma} \\ &= \lambda^x a^{1-\sigma}, \end{aligned} \tag{A.16}$$

and similarly for firm level affiliate sales

$$\begin{aligned} x^x &= \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} A(a^{1-\sigma}(\tau)^{\eta(1-\sigma)} \\ &= \underbrace{\lambda^x\left(\tau^{(1-\eta)(1-\sigma)}\right)^{-1}}_{=\lambda^m} a^{1-\sigma}. \end{aligned} \tag{A.17}$$

Then since the Pareto distribution assumption implies that $G'(a) = k(a)^{k-1}$, we can
rewrite the aggregate export sales in the following way:

$$\begin{aligned} X^x &= n\int_{a^m}^a x^x dG(a) \\ &= n\int_{a^m}^a \lambda^x a^{1-\sigma} ka^{k-1} da \\ &= n(k/(k-\sigma+1)\left[\lambda^x a^{k-\sigma+1} - \lambda^x(a^m)^{k-\sigma+1}(a^m)^k\right] \\ &= \frac{n}{k-\sigma+1}x^x G'(a)a - \frac{n}{k-\sigma+1}x^m\tau^{(1-\eta)(1-\sigma)}G'(a^m)a^m \end{aligned} \tag{A.18}$$

where we used the relationship between λ^x and λ^m highlighted in equation (A.17). Then using equation (A.18), we can derive the elasticity of the extensive margin of export:

$$
\begin{aligned}
\varepsilon_{E,\tau}^x &= -\frac{\tau}{X^x} n \left[x^x G'(a) \frac{\partial a}{\partial \tau} - x^m G'\left(a^m\right) \frac{\partial a^m}{\partial \tau} \right] \\
&= -\frac{\tau}{X^x} n \left[x^x G'(a) \left(-\frac{a}{\tau} \right) - x^m G'\left(a^m\right) \frac{\Gamma}{\tau} \right].
\end{aligned}
\tag{A.19}
$$

then since

$$
n x^m G'\left(a^m\right) a^m = (k - \sigma + 1) X^m,
\tag{A.20}
$$

The above expression allows us to rewrite equation (A.18) to obtain:

$$
X^x + \frac{n}{k - \sigma + 1} x^m \tau^{(1-\eta)(1-\sigma)} G'(a^m) a^m = \frac{n}{k - \sigma + 1} x^x G'(a) a
\tag{A.21}
$$

and using equation (A.34) we find:

$$
n x^x G'(a) a = (k - \sigma + 1) \left[X^x + \tau^{(1-\eta)(1-\sigma)} X^m \right].
\tag{A.22}
$$

The expressions in (A.22) can now be plugged in equation (A.19), to find a more compact expression for $\varepsilon_{E,\tau}^x$. This yields:

$$
\begin{aligned}
\varepsilon_{E,\tau}^x &= -\frac{\tau}{X^x} \left[(k - \sigma + 1) \left[X^x + \tau^{(1-\eta)(1-\sigma)} X^m \right] \left(-\frac{1}{\tau} \right) \right. \\
&\quad \left. -\frac{k - \sigma + 1}{k} X^m \frac{\tau}{\tau} \frac{(1-\sigma)(\eta \tau^{(1-\sigma)\eta - 1} - \tau^{-\sigma})}{\tau^{(1-\sigma)\eta} - \tau^{(1-\sigma)}} \right] \\
&= -(k - \sigma + 1) \left[\frac{X^m}{X^x} (\Gamma - \omega) - 1 \right],
\end{aligned}
\tag{A.23}
$$

where we used

$$\Gamma = \frac{\eta\tau^{(1-\sigma)\eta} - \tau^{(1-\sigma)}}{b(\tau^{(1-\sigma)\eta} - \tau^{(1-\sigma)})}, \tag{A.24}$$

$$\omega = \tau^{(1-\eta)(1-\sigma)}. \tag{A.25}$$

Notice that $\Gamma > \omega$ is true for certain parameter restrictions consistent with our calibration exercise.

We can conclude that

$$\text{if } X^m > X^x \longrightarrow \varepsilon^x_{E,\tau} < 0, \tag{A.26}$$

$$\text{if } X^m < X^x \longrightarrow \varepsilon^x_{E,\tau} > 0. \tag{A.27}$$

Finally, combining (A.13) with (A.23) gives equation (4.2).

E. Intensive and Extensive Margins of Affiliate Sales

1) Rearranging the definition of intensive and extensive margins of affiliate sales in equation (4.5), we get

$$-\frac{\partial X^m}{\partial \tau}\frac{\tau}{X^m} = \underbrace{-\frac{\tau}{X^m}\left(n\int_0^{a^m}\frac{\partial x^m}{\partial \tau}dG(a)\right)}_{\text{Intensive Margin Elasticity}} - \underbrace{\frac{\tau}{X^m}\left(nx^m G'\left(a^m\right)\frac{\partial a^m}{\partial \tau}\right)}_{\text{Extensive Margin Elasticity}}. \tag{A.28}$$

Using the definition of equilibrium individual affiliate sales, which is:

$$x^m = p^m q^m = \frac{\sigma}{\sigma-1}a\tau^\eta\beta E\frac{(a\tau^\eta\frac{\sigma}{\sigma-1})^{-\sigma}}{P^{1-\sigma}} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}A(a^{1-\sigma}(\tau)^{\eta(1-\sigma)}, \tag{A.29}$$

and considering only the partial elasticity we get

$$\frac{\partial x^m}{\partial \tau} = \eta (1 - \sigma) \frac{x^m}{\tau}.$$

(A.30)

Therefore, the elasticity of the intensive margin of affiliate sales with respect to the variable costs is:

$$
\begin{aligned}
\varepsilon_{I,\tau}^m &= -\frac{\tau}{X^m} \left(n \int_0^{a^m} \frac{\partial x^m}{\partial \tau} dG(a) \right) \\
&= \eta (\sigma - 1).
\end{aligned}
$$

(A.31)

2) Using the definition of the equilibrium productivity threshold from (3.4), we find:

$$\frac{\partial a^m}{\partial \tau} = \frac{a_m}{k} \frac{(1 - \sigma)(\eta \tau^{(1-\sigma)\eta - 1} - \tau^{(-\sigma)})}{\tau^{(1-\sigma)\eta} - \tau^{(1-\sigma)}}$$

(A.32)

We now rewrite the equation for firm level affiliate sales in (A.29), as

$$x^m = \lambda^m a^{1-\sigma}.$$

(A.33)

Then, since the Pareto distribution assumption implies that $G'(a) = ka^{k-1}$, the aggregate affiliate sales equation becomes:

$$
\begin{aligned}
X^m &= n \int_0^{a^m} x^m dG(a) \\
&= nx^m G'(a^m) \frac{a^m}{k - \sigma + 1},
\end{aligned}
$$

(A.34)

where we used the fact that $a^m G'(a^m) = k(a^m)^k$. Using equation (A.34), we can find a

solution for the elasticity of the extensive margin:

$$
\begin{aligned}
\varepsilon_{E,\tau}^m &= -\frac{\tau}{X^m}\left(nx^m G'\left(a^m\right)\frac{\partial a^m}{\partial \tau}\right) \\
&= \frac{(k-\sigma+1)}{k}\frac{(\sigma-1)(\eta\tau^{(1-\sigma)\eta}-\tau^{(1-\sigma)})}{\tau^{(1-\sigma)\eta}-\tau^{(1-\sigma)}}.
\end{aligned}
\tag{A.35}
$$

Therefore, putting together (A.31) and (A.35) gives (4.6).

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