UMR 8184

Thema

THéorie Économique, Modélisation et Applications

THEMA Working Paper n°2018-13
Université de Cergy-Pontoise, France

International business cycles: Information matters

Eleni Iliopulos, Erica Perego, Thepthida Sopraseuth

UNIVERSITÉ de Cergy-Pontoise

CNRS CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

November 2018

International business cycles: Information matters

Eleni Iliopulos[*] Erica Perego[†] Thepthida Sopraseuth[‡§]

November 2018

Abstract

We study the international transmission of shocks when agents form expectations under adaptive learning and imperfect information. To this aim we consider a two-country model featuring financial frictions, nominal rigidities, learning and Home information bias (as a source of information imperfection). We show that the more pronounced the Home information bias, the less agents track the international transmission of shocks, as it would otherwise be the case under rational expectations. The model succeeds in matching the low business cycle synchronization of consumption, while generating a positive output co-movement. In doing so, the model takes the theory closer to the data with respect to the output-consumption co-movement anomaly. The model also exhibits departure from the Uncovered Interest rate Parity.

JEL Classification : D84, E44, E51, F41, F42

Keywords: financial frictions, interdependent economies, learning, uncovered interest rate parity.

[*]EPEE, Université d'Evry, Université Paris-Saclay and Cepremap. eleni.iliopulos@univ-evry.fr. Corresponding Author.

[†]CEPII, Paris. erica.perego@cepii.fr

[‡]University of Cergy Pontoise (THEMA). thepthida.sopraseuth@u-cergy.fr

[§]We thank Stephane Adjemian, Guido Ascari, Paul Beaudry, Stefano Eusepi, Andrea Ferrero, Patrick Fève, Fabio Ghironi, Federico Giri, Cars Hommes, Jean Imbs, Peter Ireland, François Langot, Riccardo Lucchetti, Fabio Milani, Fabrizio Perri, Olivier Pierrard, Alberto Russo, Sergey Slobodyan, Henri Sneessens, Fabien Tripier, Grégory de Walque, Raf Wouters, Francesco Zanetti for helpful comments as well as conference participants at Macfinrobobs workshop (2016, Italy), SEM conference (Boston, 2017), AFSE annual meeting (Nice, 2017), ADRES Doctoral conference (Toulouse, 2017), Society for Computational Economics (New York, 2017), Conference in honor of H. Snessens (2018) and seminar participants at Cepremap and CEPII (Paris, 2016), EPEE (Evry, 2017), Università Politecnica delle Marche (Ancona, 2017), University of Economics, Prague (2017), National Bank of Belgium (2017), LAREFI (Bordeaux, 2017), GSBE seminar (Maastricht, 2017), University of Oxford (2018), University of Luxembourg (2018), University of Besancon (2018). The research leading to these results has received funding from the European Community's Seventh Framework Programme (FP7/2007-2013) under grant agreement Integrated Macro-Financial Modeling for Robust Policy Design (MACFINROBODS, grant no. 612796). We thank the Labex MME-DII (ANR-11-LBX-0023-01). Thepthida Sopraseuth acknowledges the financial support of the *Institut Universitaire de France*. This paper previously circulated under the title: "Macroeconomic Implications of Learning and Financial Frictions in Interdependent Economies".

1 Introduction

Explaining the international transmission of shocks has been one of the biggest challenges for macroeconomists since Backus et al. (1992) and Baxter & Crucini (1993). Indeed, standard macroeconomic models still have a hard time reproducing several stylized facts such as the output synchronization among countries and a relatively low international correlation of consumption levels (output-consumption correlation puzzle).[1] Moreover, standard open-economy DSGE models are built on the uncovered interest parity (UIP) condition, which eliminates all arbitrage opportunities among interest rates (once accounting for exchange rate expected dynamics and risk premia). This introduces another challenge, as departure from UIP have been extensively documented in the data (Engel (2016)).

In this paper, we explore the role of imperfect information for the behavior of the international business cycle. To do so we use a full-fledged 2-country DSGE model with financial frictions *à la* Bernanke et al. (1999), nominal rigidities, adaptive learning as in Evans & Honkapohja (2001) and imperfect information. The model brings theory closer to the data as it exhibits departure from UIP *as well as* output co-movement and low business cycle synchronization of consumption with respect to output, thereby proposing a new explanation to the output-consumption correlation puzzle.

The model builds on Faia (2007*a*) that shows that, under rational expectations, a model with nominal rigidities and financial frictions *à la* Bernanke et al. (1999) generates international financial spillovers such that output cyclical synchronization is large and positive, consistently with the data. In her model, UIP holds and is key for the international transmission mechanism. Indeed, changes in nominal interest rate in one country –that determines credit costs and play a great role in the functioning of the financial accelerator– are passed on to the other country through the UIP condition. This triggers the financial accelerator abroad that boosts output. Our contribution with respect to Faia (2007*a*) is to propose a model i) entailing low cyclical international synchronization of consumption and ii) a value of cross-country consumption correlation positive but lower than the one of output and iii) that can also account for departure from UIP. In order to obtain these features, we introduce imperfect information and adaptive learning. We show that these elements, combined with the financial accelerator, are quantitatively important to understand international business cycles and UIP dynamics.

When two economies are connected and exposed to one another business cycles, shocks are easily transmitted and agents need to form expectations based not only on the behavior of the domestic economy but also on the dynamics abroad before taking consumption and investment decisions. In our two-country model with adaptive learning, agents have an imperfect knowledge of the global

[1] Among others, recent examples are Kollman (2016), who introduces recursive preferences and uncertainty shocks on the dynamics of macroeconomic aggregates; Bai & Rios-Rull (2015), who solve the Backus-Smith puzzle by using goods market frictions and demand shocks and Eaton et al. (2016) who revisit the puzzle with a quantitative assessment of the puzzles.

economy as i) their forecasts are based on a sub-sample of the information available under RE and ii) they are updating their forecasts based on learning. In such a framework, when a shock hits, agents face a signal extraction problem that can be more or less severe depending on the amount of information at their disposal. An extreme case is when agent's forecasts are based only on information on the *local* economic environment. We will refer to this case as "Home information bias".[2] In such a framework, Home (Foreign) households do not track the impact of Foreign (Home) variables on Home (Foreign) dynamics and this affects Home and Foreign expectations. As expectations are self-fulfilling[3], the international transmission of shocks is affected. This mechanism is at the roots of both deviations from UIP and the lower synchronization of consumption with respect to rational expectation.

Expectations play a major role in financial markets. When a part of the information is not available or mis-read, financial investors can have mistaken expectations about interest rate differentials and exchange rate adjustments, and UIP does not hold. Consumers are also particularly concerned as, on the basis of their expectations, they adjust their intertemporal consumption path. If consumers under-estimate or mis-understand the transmission of a shock, they would react in a smaller (or different) way to the saving/investment needs of the other economy.

With both monetary and technological shocks as in Faia (2007*a*), the model matches the positive international output co-movement together with the lower international correlation of consumption with respect to the one of output, as in the data. In addition, by using simulated data, we regress realized exchange rate changes on international interest-rate differentials. Under rational expectations, the estimated coefficient is 1, versus 0 in the data (US-Euro Area). Under learning, the model explains approximately 20% of the departure from UIP. The sensitivity analysis confirms that limited information is key in generating the results, more than the learning algorithm *per se*. When we depart from the extreme case of complete Home information bias, the greater the information set, the closer departure from UIP and international co-movements get to the model's predictions under rational expectations.

The literature focuses either on output co-movements and the output-consumption international correlation puzzle assuming that UIP holds, or on the departure from UIP without considering the international business cycle. To our knowledge, this is the first paper that studies international co-movements in a setting with departure from UIP, using a 2-country model with financial frictions, learning and imperfect information.

Our paper lies therefore at the intersection of several strands of the literature. We contribute to international macroeconomics in the spirit of Backus et al. (1992), Baxter & Crucini (1993) and

[2]Under Home information bias, Home agents' information set includes Home variables, Home shocks as well as terms of trade and international bonds. Foreign agents' information set includes Foreign variables, Foreign shocks as well as terms of trade and international bonds.

[3]Eusepi & Preston (2018) emphasize how beliefs affect the actual evolution of the economy, which in turn affects beliefs. As expectation errors are propagated through the economy, they become partially self-fulfilling.

Obstfeld & Rogoff (2001) by proposing an explanation to the output-consumption international correlation puzzle based on the role of imperfect information and learning. In Backus et al. (1992), a positive technology shock determines a negative correlation of international output as resources are shifted to the more productive country reducing investments and output abroad. At the same time, the international correlation of consumption is positive and high as both domestic and foreign agents sustain an elevated consumption profile given the existence of complete markets and perfectly insurable risk. Since then, several works have shown that financial imperfections can break this mechanism. In a model with incomplete markets, Kehoe & Perri (2002) show that, when risk is not perfectly insurable, the correlation of consumption across countries decreases. Indeed, not all resources are transferred to the most productive economy, and this determines a positive international correlation of output and investment. However, despite the much lower correlation of consumption, the synchronization of consumption remains higher than the one of output. Faia (2007*a*) can also replicate the positive output correlation puzzle because of a financial spillover. Following a positive domestic productivity shock, inflation and Home interest rate falls. This is transmitted to the Foreign country so that Foreign interest rate also falls (together with the cost of loans), thereby boosting investment and asset prices abroad. This effect more than offsets the shift of resources to the most productive economy generating positive co-movements that are consistent with the data. However, in Faia (2007*a*)'s model, given the international financial opportunities for risk sharing, business cycle synchronization of consumption remains large compared to the data. Moreover, in general, all the above mentioned models are built under the assumption that UIP holds. Introducing imperfect information creates departure from UIP and pushes the international correlation of consumption further down, thereby obtaining the correct ordering with respect to the one of output, as in the data.[4]

In international finance, the impact of expectation errors on interest rate differentials and UIP has been explored by several papers (Lewis (1989), Gourinchas & Tornell (2004), Ilut (2012), among others). "Ambiguity averse agents" underestimate interest rate differentials or misperceive the source of the shock and leave arbitrage opportunities for the next periods (UIP is not satisfied). Chakraborty & Evans (2008) use a simplified exchange-rate model with adaptive learning to explain the forward premium puzzle. However, to our knowledge, none explored international synchronization dynamics and the importance of the information set in a full-fledged 2-country DSGE model.[5]

Finally, a strand of the learning literature develops models with financial frictions and learning in close economy. Rychalovska et al. (2016) shows the strong interaction between learning and financial frictions as agents' mis-perception of asset prices magnifies the financial accelerator. In Pintus &

[4]In this paper, we investigate how the interaction between standard financial frictions *à la* Bernanke et al. (1999) and limited information bring the model closer to the data with respect to international co-movement and departure from UIP. The study of other forms of financial imperfections (such as incomplete markets in Kehoe & Perri (2002)) combined with limited information is left for future research.

[5]Gabaix & Maggiori (2015) accounts for the failure of UIP in a stylized model with a focus on international financial investors. Tille & van Wincoop (2014) highlight the implications of information dispersion for international capital flows in a stylized general equilibrium setting.

Suda (2018) agents misperceive the leverage ratio and the response of output is amplified under learning with respect to the case of rational expectations. These papers suggest that, in a world with financial frictions, imperfect information and learning do affect macroeconomic dynamics. We extend this strand of literature by studying the international business cycle implications of learning and financial frictions.

The paper is organized as follows. We describe the model with rational expectations in Section 2 and then the one with learning (Section 3). We investigate the macroeconomic implications of learning on Impulse Response Functions (hereafter IRFs) and simulation results (Section 4). Section 5 presents the sensitivity and Section 6 concludes.

2 A two-country model with financial frictions

In this paper, we study two interconnected economies characterized both by imperfect international risk sharing and domestic financial frictions due to a costly-state verification problem. To this purpose, we build a two-country version of Bernanke et al. (1999)'s model.[6] Our benchmark model tracks the amplification mechanisms associated to the financial accelerator and the international transmission of shocks.

Each country is populated by representative households whose members receive both revenues arising from labor work in wholesale firms and profits coming from their retail activity. Households have access to international markets where they can invest in international bonds (or get indebted); they can also lend their savings to domestic (foreign) banks. As in Bernanke et al. (1999), each economy is also populated by entrepreneurs, who produce capital and decide over investment and labor inputs so as to produce wholesale goods. Capital production is affected by capital adjustment costs. To finance their production activity domestic (foreign) entrepreneurs have access to loans from domestic (foreign) banks. However, as their activity is affected both by aggregate and idiosyncratic risk, banks cannot observe their profits. In every period, a share of existing entrepreneurs defaults and exits from the market. The bank faces thus a costly-state-verification problem: in case the entrepreneur declares default, it needs to engage in a (costly) monitoring activity. Therefore, the interest rate on loans paid by entrepreneurs is greater than the one at which deposits are remunerated. There is thus a spread between lending and borrowing rates, which is carried by entrepreneurs as a risk premium. Once all production uncertainty is solved, retailers aggregate wholesale goods and sell (export) the final good to domestic (foreign) consumers. Retailers are monopolistic competitors, and their activity is affected by price rigidities à la Rotemberg. In each country rigidities affect the domestic retailing activity only (i.e. Home retailers in the Home country and Foreign retailers in the Foreign country, respectively), the exchange-rate pass through between countries is complete.

[6] See Faia (2007*a*) and Faia (2007*b*) among others.

In what follows, we will focus on the main features of our benchmark model. Notice that starred variables refer to the Foreign country. For simplicity, we will denote by H the Home country and by F the Foreign one. Our calibration will be based on US and Euro Area data, two large economies with floating exchange rate.

2.1 Households

Households in country H maximize the following flow of expected utilities

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

where β is the discount rate, C_t denotes aggregate consumption and N_t labor. The utility function $U(C_t, N_t)$ verifies the standard properties, $U_c^{'} > 0, U_c^{''} < 0, U_N^{'} < 0$, where $U_c^{'}$ is the marginal utility of consumption and $U_N^{'}$ is the marginal (dis)utility of labor effort. Aggregate consumption includes domestically produced goods and foreign ones, i.e.:

$$C = \left[(1-\gamma) C_H^{\frac{\eta-1}{\eta}} + \gamma C_F^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

with $\gamma < 0.5$ because of home bias (agents prefer domestically produced goods) and $\eta > 1$ is the elasticity of substitution between domestic and foreign goods. The CES-related CPI price index is:

$$P = \left[(1-\gamma) P_H^{1-\eta} + \gamma P_F^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

where P_H is the price of domestically-produced goods and P_F the one of foreign ones (in domestic currency). Agents' budget constraint can be written in real terms of domestic goods as[7]:

$$C_t + d_t + b_t^* \leq R_{t-1} \frac{d_{t-1}}{\pi_t} + R_{t-1}^F \frac{b_{t-1}^*}{\pi_t} \frac{e_t}{e_{t-1}} + \frac{W_t}{P_t} N_t + \frac{\Pi_t}{P_t} \tag{1}$$

where d are households' deposits in the local bank, R is the deposit rate, R^F is the return received (paid) on foreign-denominated international bonds (debt) b^*. We denote by e the nominal exchange rate (ie, the price of the foreign currency) and π is CPI inflation. Home Households' resources come from labor activity in wholesale firms and profits arising from the retailing activity. Households consume, lend funds to (perfectly competitive) banks and invest in international imperfect markets.

[7] The budget constraint in nominal terms writes as:

$$P_t C_t + D_t + B_t^* e_t \leq R_{t-1} D_{t-1} + R_{t-1}^F B_{t-1}^* e_t + W_t N_t + \Pi_t$$

where P is domestic CPI and all capital letters are written in nominal terms. Therefore, international bonds in real terms of domestic consumption can be written as $b_t^* = e_t B_t^* / P_t$.

The first order conditions of their problem (in real terms of the domestic good) read as:

$$U'_{Nt} + U'_{ct}\frac{W_t}{P_t} = 0 \tag{2}$$

$$U'_{ct} = \beta E_t\left[\frac{R_t}{\pi_{t+1}}U'_{ct+1}\right] \tag{3}$$

$$U'_{ct} = \beta E_t\left[R^F_t U'_{ct+1}\frac{e_{t+1}}{\pi_{t+1}e_t}\right] \tag{4}$$

where $\frac{W}{P}$ are real wages. Equation (2) is the optimality condition associated to labor effort. Equation (3) is the standard Euler equation associated to domestic deposits and equation (4) is the optimality equation associated to international bonds.

Due to a risk premium associated to debt accumulation, there is a spread between the return on international securities received (paid) by domestic agents and the one paid (received) by foreign ones. In particular, following Schmitt-Grohe & Uribe (2003), the spread is a function of the (real) value of the country's net external debt so that the interest rate on international bonds is defined as:

$$R^F_t = R^*_t + p\left(-b^*_t\right) \tag{5}$$

where R^* is the foreign nominal interest rate and $p\left(-b^*_t\right) = -\zeta\left(exp^{b^*_t - b^*} - 1\right)$ a country-specific interest rate premium with $\zeta > 0$. Foreign households face the same optimization problem as domestic households except for the fact that international bonds are denominated in their own currency. By combining agents' Euler equations we obtain the following uncovered interest parity condition:

$$U'_{ct} = \beta E_t\left[\left(R^*_t + p\left(-b^*_t\right)\right)U'_{ct+1}\frac{e_{t+1}}{\pi_{t+1}e_t}\right]$$

and thus:

$$U'_{ct} = \beta E_t\left[\left(\frac{U^{*\prime}_{ct}}{\beta E_t\left[\frac{U^{*\prime}_{ct+1}}{\pi^*_{t+1}}\right]} + p\left(-b^*_t\right)\right)U'_{ct+1}\frac{e_{t+1}}{\pi_{t+1}e_t}\right] \tag{6}$$

so that marginal utilities across countries are equalized up to a spread for the country risk. Notice finally that terms of trade for the Home country are the ratio of the price of domestic goods over the price of foreign goods, $tot_t = \frac{P_{Ht}}{e_t P^*_{Ft}} = \frac{f_t}{e^r_t f^*_t}$, where $f_t \equiv \frac{P_{Ht}}{P_t} = f_{t-1}\frac{\pi_{Ht}}{\pi_t}$ and $f^*_t \equiv \frac{P^*_{Ft}}{P^*_t} = f^*_{t-1}\frac{\pi^*_{Ft}}{\pi^*_t}$.

2.2 Entrepreneurs

We now focus on domestic entrepreneurs (F entrepreneurs' optimization problem is symmetric). As in Bernanke et al. (1999), entrepreneurs are risk neutral and choose the optimal level of both capital and labor inputs to be used for wholesale production. Once idiosyncratic uncertainty is solved, wholesale output is:

$$Y_t = A_t F\left(K_{t-1}, N_t\right)$$

where K denotes capital, N labor and A the exogenous total factor productivity:

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^{as}.$$

with $0 < \rho_A < 1$ and ε_t^{as} the productivity shock with standard deviation σ_A. Capital evolves as:

$$K_t = (1-\delta)K_{t-1} + I_t$$

where δ is the depreciation rate and I investment. The optimality condition with respect to labor is:

$$f_t \frac{Y_{N,t}}{X_t} = \frac{W_t}{P_t}$$

where $Y_{N,t}$ denotes the first derivative of output w.r.t. labor and X_t the gross markup of retail goods over wholesale goods (i.e. $\frac{1}{X_t} = \frac{P^w}{P_H}$ where, in turn, P^w is the wholesale output price and P_H is the price of the domestic production). The optimal investment decision verifies:

$$Q_t = \left[1 + \Phi'\left(\frac{I_t}{K_{t-1}}\right) K_{t-1}\right]$$

where Q_t is the (real) price of capital and it is different from one around the steady-state because of capital adjustment costs. The mean return from holding one unit of capital is thus:

$$R_t^k = \frac{\pi_t}{Q_{t-1}} \left[\frac{Y_{K,t-1}}{X_t} f_t + \Phi'\left(\frac{I_t}{K_{t-1}}\right)\frac{I_t}{K_{t-1}} - \Phi\left(\frac{I_t}{K_{t-1}}\right) + Q_t(1-\delta)\right] \tag{7}$$

where the first term in the brackets represents the domestic-currency yields of one unit of capital, $\frac{Y_{K,t-1}}{X_t} f_t$($Y_{K,t-1}$is the derivative of output w.r.t. capital); the second one is the reduction in adjustment costs, $\Phi'\left(\frac{I_t}{K_{t-1}}\right)\frac{1}{K_{t-1}} - \Phi\left(\frac{I_t}{K_{t-1}}\right)$; and the third term captures the returns from selling that unit of non-depreciated capital, $Q_t(1-\delta)$.

2.3 Loan contract and wealth accumulation

In each period t, a continuum of entrepreneurs (indexed by j) needs to finance the purchase of new capital K_t^j that will be used for production in period $t+1$. The entrepreneur engages in a financial contract *before* the realization of the idiosyncratic shock, ω^j. Indeed, at the moment in which the contract is signed, both the bank and the entrepreneur do not know the rate of return of capital, $\omega^j R^k$. Each period, each entrepreneur owns end-of-period internal funds for an amount nw_t^j (in real terms of the consumption good). As in Bernanke et al. (1999), we assume that the required funds for investment exceed internal funds, and thus:

$$l_t^j = Q_t K_t^j - nw_t^j \geqslant 0 \tag{8}$$

where l_t^j denotes the loans needed by entrepreneur j to finance investment projects. Default occurs when the return from the investment $\omega_{t+1}^j R_{t+1}^k Q_t K_t^j$ is lower than the amount that needs to be repaid $R_t^L l_t^j$, i.e.,

$$\omega_{t+1}^j \leq \tilde{\omega}_{t+1}^j \equiv \frac{R_t^L l_t^j}{R_{t+1}^k Q_t K_t^j}$$

where $\tilde{\omega}^j$ is the threshold level for the productivity idiosyncratic shock below which entrepreneurs default. We denote by R^L the borrowing rate paid by entrepreneurs. We recall that, as Bernanke et al. (1999), the borrowing rate R^L is an endogenous result of the optimal debt contract proposed by banks to entrepreneurs. Indeed, the bank knows that the entrepreneur has an incentive to declare default so as not to pay back its debt. As shocks are specific to each entrepreneur, j, each time s/he declares default, the bank needs to engage in a monitoring activity. As in Bernanke et al. (1999), we suppose that in each period only a fraction of entrepreneurs survives while the other fraction defaults and exits the market. It is possible to rewrite banks' net capital output share as a function of the threshold default level, ω^j:

$$\Gamma\left(\tilde{\omega}_{t+1}^j\right) = \int_0^{\tilde{\omega}_{t+1}^j} \omega_{t+1}^j f(\omega) d\omega + \tilde{\omega}_{t+1}^j \int_{\tilde{\omega}_{t+1}^j}^{\infty} f(\omega) d\omega$$

and the implied monitoring cost share:

$$\mu G\left(\tilde{\omega}_{t+1}^j\right) \equiv \mu \int_0^{\tilde{\omega}_{t+1}^j} \omega_{t+1}^j f(\omega) d\omega$$

so that the optimal contract results from maximizing banks' expected real profits:

$$E_t\left\{\left[1 - \Gamma\left(\tilde{\omega}_{t+1}^j\right)\right] R_{t+1}^k Q_t K_t^j\right\}$$

under the bank participation constraint

$$\left[\Gamma\left(\tilde{\omega}_{t+1}^j\right) - \mu G\left(\tilde{\omega}_{t+1}^j\right)\right] R_{t+1}^k Q_t K_t^j = R_t\left(Q_t K_t^j - nw_t^j\right) \tag{9}$$

which implies zero profits. Because i) only a share of entrepreneurs remains alive in every period and ii) both the cut-off value and the external finance premium are linear with respect to the capital-wealth ratio, aggregation across entrepreneurs is possible. By aggregating wealth, the optimality condition resulting from the bank optimal program can be rewritten as:

$$E_t \frac{R^k_{t+1}}{R_t} = E_t \frac{1}{\frac{[1-\Gamma(\tilde{\omega}_{t+1})][\Gamma'(\tilde{\omega}_{t+1})-\mu G'(\tilde{\omega}_{t+1})]}{\Gamma'(\tilde{\omega}_{t+1})} + [\Gamma(\tilde{\omega}_{t+1}) - \mu G(\tilde{\omega}_{t+1})]} \tag{10}$$

or

$$E_t \frac{R^k_{t+1}}{R_t} = \rho(\tilde{\omega}_{t+1}) \tag{11}$$

where $\rho'(\tilde{\omega}) \succeq 0.\rho(\tilde{\omega})$ is the external finance premium. The ratio $E_t \frac{R^k_{t+1}}{R_t}$ captures the cost of finance, which reflects in turn the existence of monitoring costs. Using equation (9), we get

$$[\Gamma(\tilde{\omega}_{t+1}) - \mu G(\tilde{\omega}_{t+1})] \frac{R^k_{t+1}}{R_t} \frac{Q_t K_t}{nw_t} = \left(\frac{Q_t K_t}{nw_t} - 1 \right) \tag{12}$$

With equations (10) and (11), equation (12) defines a relationship between the external finance premium and the leverage ratio $\frac{Q_t K_t}{nw^j_t}$. Surviving entrepreneurs accumulate wealth. We assume that the wealth belonging to defaulting entrepreneurs is instead consumed by existing ones. Thus, the consumption level of surviving entrepreneurs is:

$$C^e_t = (1 - \varsigma_t)[1 - \Gamma(\tilde{\omega}_t)] \frac{R^k_t Q_{t-1}}{\pi_t} K_{t-1}$$

Aggregate wealth can be written as

$$nw_t = \varsigma_t R^k_t \frac{Q_{t-1}}{\pi_t} K_{t-1} - \frac{\varsigma_t}{\pi_t} \left[R_{t-1} + \frac{\mu G(\tilde{\omega}_t) R^k_t Q_{t-1} K_{t-1}}{(Q_{t-1}K_{t-1} - nw_{t-1})} \right] (Q_{t-1}K_{t-1} - nw_{t-1}) \tag{13}$$

with $\frac{\mu G(\tilde{\omega}_t) R^k_t Q_{t-1} K_{t-1}}{(Q_{t-1}K_{t-1} - nw_{t-1})}$ the risk premium factor.

2.4 Final good production

As in Bernanke et al. (1999), Home retailers aggregate wholesale goods to the purpose of producing final goods X^c according to the following Dixit-Stiglitz aggregator, $X^c = \left(\int_0^1 X^c(i)^{\frac{\upsilon-1}{\upsilon}} di \right)^{\frac{\upsilon}{\upsilon-1}}$, with $\upsilon > 0$ the elasticity of substitution between domestic varieties. They operate in a monopolistic competition framework and price setting is affected by nominal rigidities à la Rotemberg with quadratic price adjustment costs $\frac{\omega_P}{2}(\pi_{Ht} - 1)^2$, where π_H denotes producer price inflation in country H and $\omega_P > 0$ is the Rotemberg parameter for price rigidity. Retailers' optimization problem leads to the following Phillips curve:

$$(\pi_{Ht} - 1)\pi_{Ht} = Y_t \frac{\upsilon}{\omega_P} \left[\frac{1}{X_t} - \frac{(\upsilon - 1)}{\upsilon} \right] + \beta E_t \left[\frac{U'_{ct+1}}{U'_{ct}} (\pi_{Ht+1} - 1) \frac{f_{t+1}}{f_t} \pi_{Ht+1} \right] \tag{14}$$

Analogously, country F retailers' problem entails the following Phillips curve:

$$\left(\pi^*_{Ft}-1\right)\pi^*_{Ft}=Y^*_t\frac{\upsilon}{\omega_P}\left[\frac{(1-\upsilon)}{\upsilon}+\frac{1}{X^*_t}\right]+\beta E_t\left[\frac{U^{*\prime}_{ct+1}}{U^{*\prime}_{ct}}\left(\pi^*_{Ft+1}-1\right)\frac{f^*_{t+1}}{f^*_t}\pi^*_{Ft+1}\right]$$

where π^*_F denotes producer price inflation in country F.

2.5 Monetary policy

We suppose that in each country the monetary policy follows empirical Taylor rules. Therefore, the monetary rule in country H is:

$$R_t=\left(R_{t-1}\right)^{\chi}\left(\bar{R}^n\left(\frac{\pi_t}{\bar{\pi}}\right)^{b_\pi}\left(\frac{y_t}{y}\right)^{b_y}\right)^{1-\chi}mp_t \tag{15}$$

In country F,

$$R^{*n}_t=\left(R^{*n}_{t-1}\right)^{\chi^*}\left(\bar{R}^{*n}\left(\frac{\pi^*_t}{\bar{\pi}^*}\right)^{b^*_\pi}\left(\frac{y^*_t}{y^*}\right)^{b^*_y}\right)^{1-\chi}mp^*_t \tag{16}$$

with a mp_t and mp^*_t temporary monetary policy shocks, such that:

$$\log mp_t = \rho_{mp}\log mp_{t-1}+\varepsilon^{mp}_t \tag{17}$$

$$\log mp^*_t = \rho_{mp}\log mp^*_{t-1}+\varepsilon^{mp*}_t \tag{18}$$

with $0<\rho_{mp}<1$ and standard deviation of monetary innovations denoted σ_{mp}.

2.6 Calibration

Each period corresponds to one quarter. The calibration of this model is mostly based on the works of Faia (2007*a*), Christiano et al. (2014) (hereafter, CMR) and Kolasa & Lombardo (2014) (hereafter, KL). We assume that the Home country is the Euro area and the Foreign country refers to the US. Table 1 summarizes the calibration.

Preferences: We let the instantaneous utility function be $U_t=\frac{C_t^{1-\sigma}}{1-\sigma}+\Psi\log\left(1-N_t\right)$. The intertemporal elasticity of substitution for consumption is set in both countries equal to 2, consistently with the literature. The disutility of the labor parameter is set -in both countries- equal to 2.6 so as to insure that labor is normalized to 1/3 at steady state. The discount factor is in both countries equal to 1/1.01147, consistently with CMR annual interest rate. The share of foreign goods into the domestic basket, γ, is equal to 0.4, as in KL and the elasticity of substitution between foreign vs domestic goods is 1.5 as in Faia (2007*a*). The elasticity of substitution among varieties υ is set equal to 6 as in CMR (among others).

Production: The wholesale production function is a Cobb-Douglas, $Y_t = a_t K_t^{\alpha} N_t^{1-\alpha}$ where α is set to 0.36 and the capital depreciation rate is 0.025 as in CMR among others. The capital adjustment costs parameter Φ is set to 5.2 in both countries as in CMR. The Rotemberg parameters are calculated both for the EU and the US as in Monacelli (2009) starting from CMR estimates of the Calvo parameters in the EU and the US (around 0.7 and 0.6, respectively).

Financial parameters: The monitoring cost parameter, μ is set to the same level in both countries in order to keep the model as symmetric as possible. We let μ =0.21 in both countries, based on CMR. The interest rate premium parameter, $\zeta = 0.000742$ as in Schmitt-Grohe & Uribe (2003). We set the share of surviving entrepreneurs $\varsigma = 0.978$, consistently with CMR as well.

Monetary policy: The weight on inflation and output in the Home Taylor rule is set as in KL with $b_\pi = 2.4$ and $b_y = 0.15$. In the Foreign country, the same parameters are fixed consistently with the estimates of CMR on US data. We let $b_\pi^* = 2.6$ and $b_y^* = 0.36$. In both countries $\chi = 0.8$ in line with Faia (2007a).

Shocks: All shocks are log-normal AR(1) and calibrated following Faia (2007a). The persistence parameter of the productivity shock is set in both countries to 0.8 while the standard deviation of ε_t^{as} is set to 0.008. At steady state $A = 1$ in both countries. In contrast to Faia (2007a), we do not assume cross-country correlation of technological innovations in order to make the interpretation of economic mechanisms more straightforward. Finally, the standard deviation of the monetary shock is set to 0.005 and the autoregressive parameter to 0.0001.

3 Learning

3.1 Adaptive learning

As mentioned by Sims (1980), there are many ways of modeling non-rational behaviors. In this paper, private agents engage in adaptive learning following the standard methodology put forward by Evans & Honkapohja (2001). This provides a useful starting point to compare our work to the literature. The model is approximated at order 1, as in Evans & Honkapohja (2001), and the corresponding reduced form is:

$$k_t = a_1 E_t k_{t+1} + a_2 k_{t-1} + b_1 z_t + b_2 z_{t-1} \tag{19}$$

$$z_t = \rho z_{t-1} + \varepsilon_t \tag{20}$$

with z_t the vector of shocks and k_t a vector of all endogenous variables in the model. Private agents have beliefs on the evolution of macroeconomic variables in the economy, based on their Perceived

Table 1: Calibration

Parameter		Value	Reference
β	discount factor	0.9887	CMR (2014)
σ	elasticity of intertemporal substitution	2	KL (2014)
Ψ	disutility of labor	2.6	Labor normalized at 1/3 at ss
γ	share of foreign goods in domestic basket	0.4	KL (2014)
η	elasticity of substitution between home and foreign goods	1.5	Faia (2007*a*)
υ	elasticity of substitution between varieties	6	CMR (2014)
α	production function	0.36	CMR (2014)
δ	capital depreciation	0.25	CMR (2014)
ϕ	capital adjustment costs	5.2	CMR (2014)
ω_p	Rotemberg parameter	41.75	CMR (2014), Monacelli (2009)
ω_{p*}	Rotemberg parameter	35.8467	CMR (2014), Monacelli (2009)
μ	monitoring cost	0.21	CMR (2014)
ζ	interest rate premium parameter	0.000742	SGU (2003)
b^*	steady state Net Foreign Asset	0	SGU (2003)
ς	share of surviving entrepreneurs	0.978	KL (2014)
χ	weight on lagged int. rate into Taylor Rule	0.8	Faia (2007*a*)
χ_*	weight on lagged int. rate into Taylor RuleTR	0.8	Faia (2007*a*)
b_π	weight on inflation into Taylor Rule	2.4	KL (2014)
$b_{\pi*}$	weight on inflation into Taylor Rule	2.6	KL (2014), CMR (2014)
b_y	weight on output gap into Taylor Rule	0.15	KL (2014)
b_{y*}	weight on output gap into Taylor Rule	0.36	KL (2014), CMR (2014)
ρ_A	persistence techno shock	0.8	Faia (2007*a*)
σ_A	sd techno shock	0.008	Faia (2007*a*)
ρ_{mp}	persistence monetary shock	0.0001	Faia (2007*a*)
σ_{mp}	sd monetary shock	0.005	Faia (2007*a*)

CMR(2014) refer to Christiano et al. (2014), KL(2014) to Kolasa & Lombardo (2014), SGU (2003) to Schmitt-Grohe & Uribe (2003)

Law of Motion (PLM):

$$k_t = \phi_{k,t-2}x_{t-1} + \phi_{z,t-2}z_{t-1} \quad (21)$$

Private agents think that endogenous variables k_t are a function of a set of observed variables x_{t-1} and exogenous shocks z_{t-1}. Private agents use the PLM to forecast economic variables.

$$E_t k_{t+1} = \phi_{k,t-1}k_t + \phi_{z,t-1}z_t \quad (22)$$

The actual evolution of macroeconomic variables in the economy is obtained by replacing the expected value from equation (22) into the reduced form (equation (19)). In doing so, it becomes clear that beliefs affect the actual dynamics of the economy, which in turn affect beliefs. This is the so-called "self-referentiality" in models with learning (Eusepi & Preston (2018)).

It is thus necessary to define: i) the set of observed variables x included in the PLM (equation (21)); ii) the methodology used to update time-varying coefficients ϕ in the PLM and iii) the initialization of ϕ. Many choices can be made at this stage. Obviously, each of these choices affect the macroeconomic behavior of the economy. We describe below the rationale behind each of our choices and present a sensitivity analysis of each of these assumptions in section 5.

3.2 Modeling choices

***i).* Information set used for adaptive learning: Home information bias.** Under rational expectations, all agents in the two countries observe all economic variables in the world. They use this wide information set in their PLM and forecasting models. In contrast, under learning, agents have an imperfect knowledge of their economic environment. They then use a reduced information set when forming their expectations. We start with an extreme case in which, in each country, private agents base their PLM only on *local* variables, including *local* shocks, together with international bonds b^* and terms of trade. Agents do not observe macroeconomic variables abroad.[8] We will refer to this assumption as Home information bias.[9] The idea is not new. In the finance literature, it is widely used either as an explanation for the difference in forecasting performances[10] or as the result of a strategic behavior.[11] The empirical evidence also suggests that local investors have an information advantage and outperform foreign investors (Bae et al. 2008, Dvorak 2005, Ferreira et al. 2017, Hau 2001, Teo 2009). In the learning literature, using reduced

[8] As only the Foreign bond (denominated in Foreign currency) is available in the model, only Home households forecast the nominal exchange rate.

[9] Unlike Orphanides & Williams (2007), central banks have access to all information. We discard any learning behavior from central banks and leave this point for future research.

[10] Home investors predict home asset payoffs better than foreigners (Dziuda & Mondria (2012)).

[11] Home investors do not learn from foreigners as they profit more from knowing information others do not know (Van Nieuwerburgh & Veldkamp (2009)).

information sets implying small forecasting models improves the model's fit to the data (Slobodyan & Wouters (2012*a*), Ormeno & Molnar (2015), Hommes et al. (2015)). In Section 5.1, we measure the sensitivity to this assumption.

ii). **Learning method.** In this work, we assume that agents update their believes using a stochastic-gradient constant-gain (using the semantics in Carceles-Poveda & Giannitsarou (2007)).

$$\phi_t \quad = \quad \phi_{t-1} + gain \times x_{t-1} \left(k_t - x'_{t-1} \phi_{t-1} \right) \tag{23}$$

Private agents form adaptive expectations: after observing the state of the economy, they correct their previous estimate of ϕ using their forecast error. Under adaptive learning, individuals behave much like econometricians, using new observations on macroeconomic conditions to update their estimates of key economic relationships. Private agents do not cease to update coefficients in their PLM, based on their forecast errors. They engage in real-time perpetual learning.

In the literature, private-agent updating parameter $gain$ lies between 0.01 and 0.05 (Orphanides & Williams (2005), W.A. & Evans (2006), Milani (2007), Slobodyan & Wouters (2012*b*)). $1/gain$ can be interpreted as an indication of how many past observations agents take into account to form their expectations. We choose $gain = 0.03$ as a benchmark value which implies that private-agents approximately use 33 quarters or 8 years of data in their forecasts. This value is consistent with the one chosen in the papers on monetary policy under learning ($gain = 0.02$ in Orphanides & Williams (2008), $gain = 0.03$ in Gaspar et al. (2011)) and with the estimates of Milani (2007) using Bayesian techniques and Orphanides & Williams (2005) exploiting data on expectations from the Survey of Professional Forecasters. In Section 5.3 we provide a robustness exercise on the value of this parameter.

iii). **Initialization of the learning loop.** Equation (23) is based on a recursive specification, which requires initialization (ϕ_0). Initialization is a crucial step as errors on initial perceptions take a long time to die out (Carceles-Poveda & Giannitsarou (2007), Slobodyan & Wouters (2012*b*)). In order to clearly study the role of learning and of the information set available to the agents, we initialize the process at rational expectations. In such a way, we remove the effect of the initialization from the dynamics. This allows us to properly compare the new results with learning to the rational expectation version of the model.

In what follows, we first consider information incompleteness when agents' expectations are initialized at rational expectations and update their beliefs based on past forecast errors. In section 5, we analyze the impact of each of these elements in order to assess their relative importance in our model. Notice that, if we assume complete information rather than Home information bias (with the initialization at rational expectations) the model could potentially behave very closely to rational expectations, depending on the learning process. We investigate this intuition in section 5.2.

4 Inspecting the economic mechanisms

4.1 Impulse response functions

In what follows, we analyze the transmission mechanisms of our economy in response to the aggregate stochastic processes, highlighting the different mechanisms at work: *a)* the effects of financial frictions; *b)* in interdependent economies with neo-Keynesian features; *c)* both under rational expectations and learning.

Point *a)* was investigated by Bernanke et al. (1999), and more recently by Christiano et al. (2011) in a small open economy setting. Our IRFs under rational expectations for the local economy in response to a domestic shock are consistent with the ones displayed in Christiano et al. (2011). Point *b)* relates to the international transmission of exogenous shocks in interdependent economies with financial frictions, which was analyzed by Kolasa & Lombardo (2014), Faia (2007*a*) and Faia (2007*b*) among others. The IRFs under rational expectations are consistent with theirs. Our contribution lies in *c)*. In what follows below we will quickly summarize the economic mechanisms under rational expectations in open-economy (*a)* and *b)*) before stressing the different macroeconomic dynamics under learning. In addition, for the sake of brevity, we describe below the effects of Home shocks, as the effects of Foreign shocks are nearly symmetric.[12]

4.1.1 Positive Home technological shock

Rational expectations (RE) .

Effects of Home shock in the Home country. The productivity shock boosts labor and capital productivity. It has then a positive impact on output (Fig. 1), capital accumulation (Fig. 2) and investment. Moreover, as expected, the positive supply shock entails a downward pressure on domestic prices and inflation that dampens in turn domestic interest rates through the Taylor rule (Fig. 3). The cost of credit falls, the (real) price of capital jumps (Fig. 2) and entrepreneurs accumulate more net wealth, which further improves their access to credit. The financial accelerator is at work. Thanks to the increased productivity, the price of wholesale production decreases so that retailers' marginal costs fall and their profits increase. Households consume more (Fig. 1). They also invest in international bonds (Fig. 3), so that the net external position improves together with the trade balance. Indeed, there is an expenditure switching effect in favor of country H following the deterioration of terms of trade in the Home country (see Fig. 11).[13]

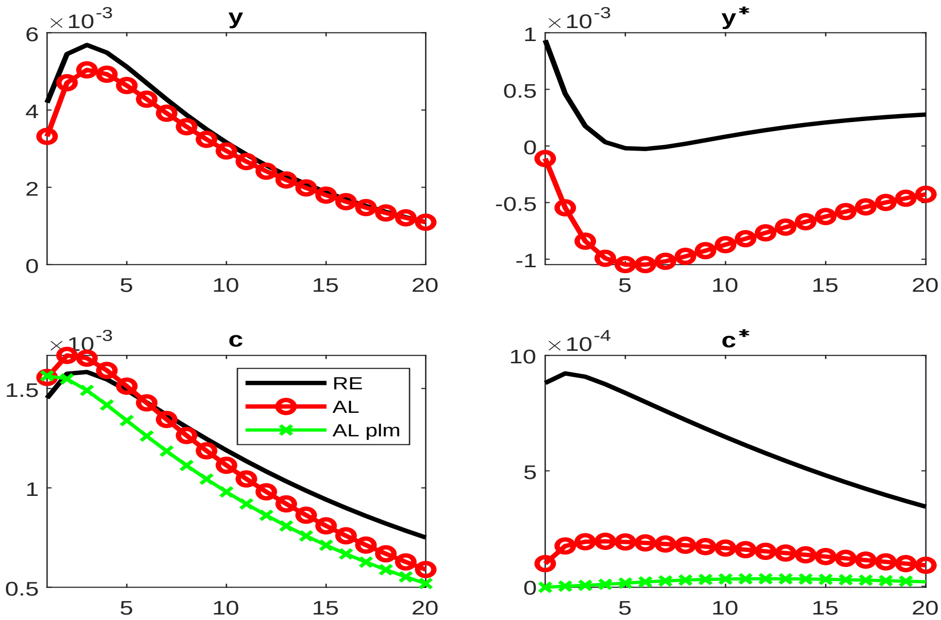
International spillovers and UIP. Foreign agents enjoy an improvement in their terms of trade. This triggers a demand shift towards domestic goods, to the detriment of Foreign goods. The weaker

[12]In the benchmark calibration, countries only differ with respect to their price rigidity and Taylor rules.

[13]We remind the reader that variable *tot* is defined as the price of the Home good relative to the price of the Foreign good in Home currency.

demand for Foreign goods results in a decrease in foreign inflation. The fall in foreign inflation induces a fall in foreign interest rates, due to the response of monetary policy coupled with sticky prices. The cost of loans abroad declines. This triggers the financial accelerator in country F. Foreign entrepreneurs experience thus a fall in the cost of loans that boosts investment and asset prices. This is the so-called "financial spillover effect" in Faia (2007*a*) that makes the model match output co-movement under rational expectations. Lower local interest rates also stimulate current Foreign consumption (Fig. 1), thereby generating high consumption correlation. As mentioned by Faia (2007*b*), the model does not solve the consumption-output anomaly: the cross correlation of output is positive but still lower than consumption correlation, which is counterfactual. Finally, UIP holds by assumption.

Figure 1: Home technological shock



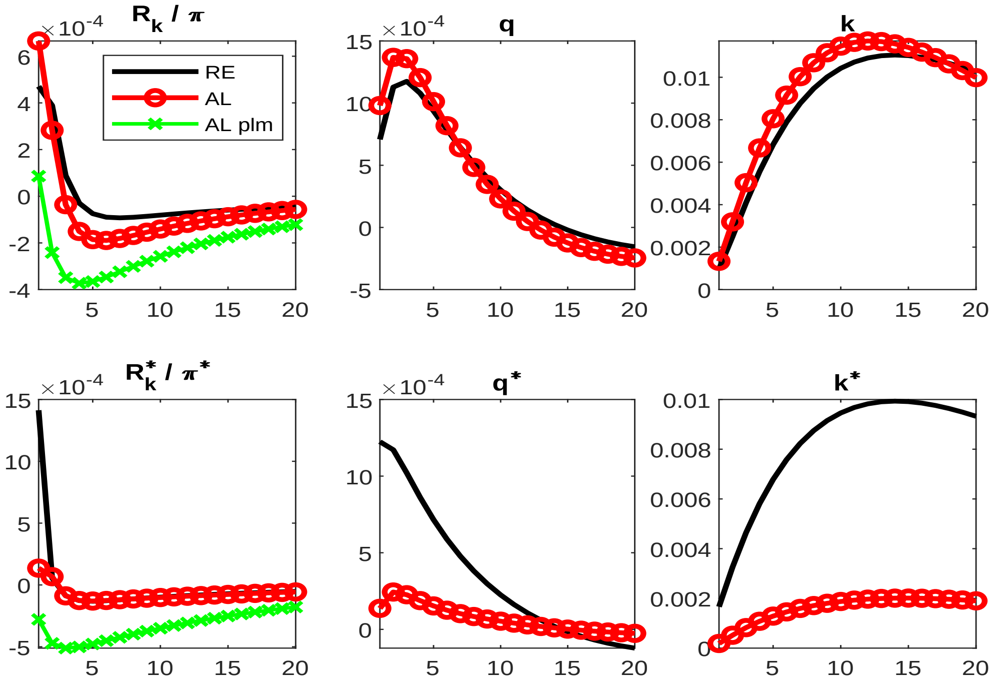
Solid line: Rational Expectations. "o" line: Adaptive learning. "+" line: Adaptive learning, $E_t[x_{t+1}]$ forecast of variable x based on PLM. Only for forward-looking variables. Deviation from steady state after one-sd positive shock.

Adaptive learning (AL) under Home information bias .

Learning affects macroeconomic dynamics through the expectation terms in the equations determining agents' intertemporal decisions. The forecasts of future inflation, consumption, entrepreneurs' productivity threshold, nominal exchange rate and real return on investment affect current behaviors i.e., household's savings, entrepreneurs' investment and lending decisions, hence price and output dynamics, which in turn affect agents' beliefs.

We report into the graphs of the IRFs the expected values $E_t[x_{t+1}]$ of forward variables $(de, \frac{R_k}{\pi}, c, \pi, \tilde{\omega})$ that affect agents' current behavior. We consider a variable as mis-perceived whenever the expected

Figure 2: Home positive technological shock



Solid line: Rational Expectations. "o" line: Adaptive learning. "+" line: Adaptive learning, $E_t[x_{t+1}]$ forecast of variable x based on PLM. Only for forward-looking variables. Deviation from steady state after one-sd positive shock.

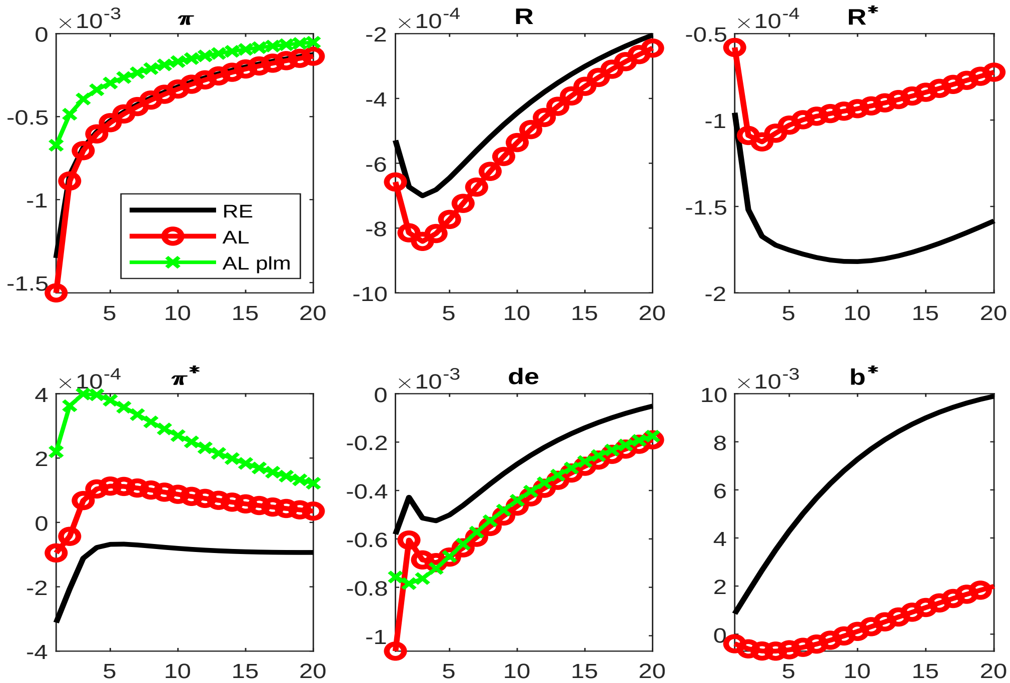
value (the forecast $E_t[x_{t+1}]$ using the *Perceived* Law of Motion) significantly departs from the realized value under learning (*Actual* Law of Motion).

Effects of Home productivity shock in the Home country. With learning, domestic agents do not take into account the positive financial spillover that boosts Foreign output, hence Home exports. They have thus more conservative expectations. In particular, Home agents think that the macroeconomic effects of the supply shock are smaller than they actually are.[14] Therefore, Home agents expect CPI inflation to decrease less than with RE (Fig. 3) and Home output is slightly lower than under RE (Fig. 1). Lower levels of output entail in turn a more accommodative monetary policy. This explains why, interestingly, in response to the shock the realized level of inflation with learning is eventually lower with respect to inflation's expectations (see the green line in Fig. 3) and with respect to the RE case.[15]

[14] Additionally, in Fig. 11, in Appendix C, both the expected and realized default threshold, $\tilde{\omega}$, are greater with learning than with RE, which denotes that agents expect financial conditions to be not as favorable as under rational expectations.

[15] As we can see from Figures 1-3, the overall effect of the *local* shock in the *local* economy under learning is not very far from the one under rational expectations. This is consistent with Carceles-Poveda & Giannitsarou (2007)'s findings in a closed economy RBC model. As in their model, agents form their 1-step ahead expectations based on the observation of all the domestic variables, including domestic shocks and initializing their believes at rational expectations. Quantitatively, for Home agents, the information on the Home shock is sufficient to make their behavior close to the one under RE. The learning algorithm affects the speed of convergence to the steady state through the adaptive process in the update of beliefs. In fact, what amplifies the effect of learning in our model is the interaction with imperfect information. The part of information the agents miss is the one coming from the other country:

Figure 3: Home positive technological shock



Solid line: Rational Expectations. "o" line: Adaptive learning. "+" line: Adaptive learning, $E_t[x_{t+1}]$ forecast of variable x based on PLM. Only for forward-looking variables. Deviation from steady state after one-sd positive shock. $de_t = e_t - e_{t-1}$ is the first difference of the nominal exchange rate. $de_t < 0$ is an appreciation of the Home currency.

International spillovers and departure from UIP. In our framework with Home information bias, Home (Foreign) households do not track the impact of Foreign (Home) variables on Home (Foreign) dynamics. Therefore, agents in the two regions do not take into account the possibility that the Foreign (Home) variables could react to the shock. The international transmission of shocks can thus significantly differ from the one under RE. This is also at the roots of quantitatively significant deviations from UIP.

In particular, Foreign agents update their expectations reacting only to the observed international capital inflows, terms of trade and local macroeconomic changes. After a Home productivity shock, Foreigners observe an improvement in their terms of trade. On the one hand, this shifts their demand away from Foreign goods, which drives Foreign inflation down. On the other hand, Foreign agents do not track the source of the shock and simply expect higher future Foreign inflation as the price of Foreign goods increases with respect to Home goods (they observe their (Home) terms of trade improving (deteriorating), see Fig. 11). Because of higher inflation expectations (Fig. 3), inflation eventually increases via the Phillips curve. Monetary policy reacts to local economic conditions by reducing the policy rate, but less than under RE, as Foreign inflation under learning is

Foreign agents do not observe Home shocks and Home agents do not take into account feedback effects coming from the Foreign country. As the domestic variables contain the majority of the relevant information after a Home shock, the Home dynamics are generally not far from the ones under RE.

now higher than under RE. As the Foreign interest rate decreases less than under RE, the financial accelerator mechanism in the Foreign country is dampened. As soon as Foreign agents receive firms' profits, consumption increases also abroad – but much less than under RE. The international correlation of consumption is therefore lower, driven by self-fulfilling agents' expectations. Because Foreign consumption and investment increase less, Foreign output is lower under learning than under rational expectations (Fig 1). Notice also that the expected increase in Foreign inflation pushes Foreign agents to switch in favor of Home goods (expenditure switching). Thus, even if consumption (slightly) increases, output initially (slightly) deteriorates. Output under learning is then less synchronized than under rational expectations after a Home technological shock.

Notice finally that the shock affects Foreign interest rates via the Taylor rule. In particular, the monetary policy reacts to self-fulfilling "wrong" inflation expectations (expectations with learning are indeed not perfect). Moreover, with imperfect information, Home (Foreign) agents do not account for the fact that Foreign (Home) agents do react to the shock and do not correct their expectations correspondingly. Home agents do not observe the Foreign interest rate response. As a result, they expect the international interest rate differential to be larger than what it actually is. Home agents hence anticipate a large Home currency appreciation, compared to the case of rational expectations. This expectation on the exchange rate is self-fulfilling. Agents, tend thus to buy less Foreign bonds (denominated in Foreign currency, see Fig 3). This dampens the demand for Foreign currency, thereby leading to a larger actual appreciation under AL (than under RE). This also explains why in response to a technological shock, the nominal exchange rate dynamics are inconsistent with UIP.

4.1.2 Monetary tightening: Positive Home monetary shock

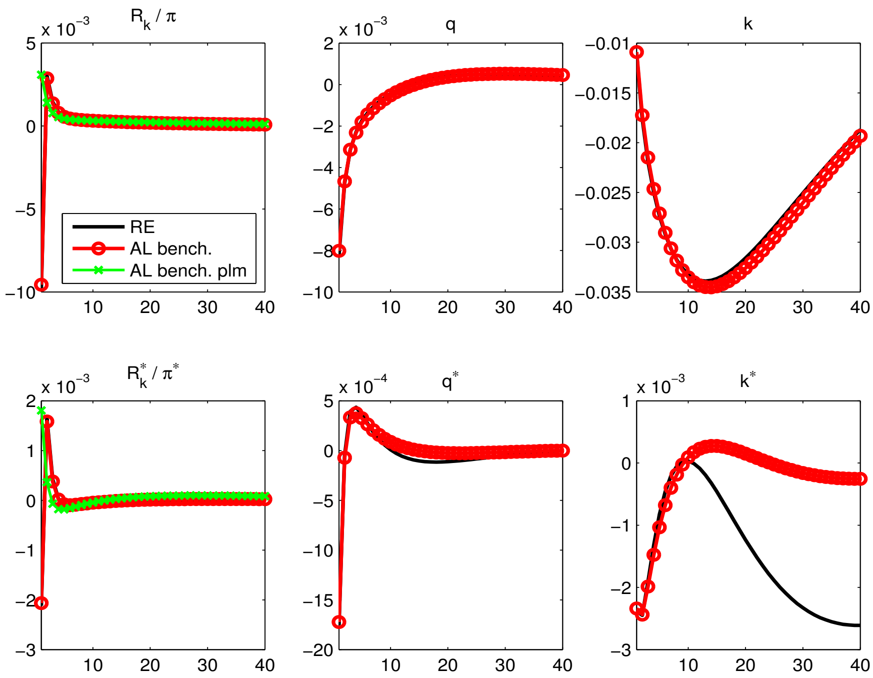
Rational expectations .

Effects of Home monetary shock in the Home country. IRFs are displayed in Figures 4-6. A monetary shock triggers a straightforward increase in the domestic interest rate (equation (17), Fig. 6) and a decrease in the domestic CPI inflation rate. The fall in inflation is however limited by price rigidities. Domestic demand is reduced both through households consumption (Fig. 5) and lower entrepreneurs' investment. This dampens in turn output production. The lower demand for capital results in a decrease in the price of capital. Net wealth goes down due to the joint effect of the fall in the price of capital and the Fisher effect, which increases the real burden of debt. Reduced net wealth, coupled with lower price of capital, further deteriorates entrepreneurs' financial conditions, which deepens the recession.

International spillovers and UIP. The contraction in domestic demand generates a trade surplus and international asset holdings of Home households increase. As domestic demand for Foreign goods falls, the exchange rate appreciates (Fig. 6), and the Home terms of trade improve (see

Fig. 11, in Appendix C). The Home currency appreciation is consistent with Kollmann (2001) and Christiano et al. (2011). The increase in interest rates is transmitted abroad through the UIP and the monetary rule with analogous recessionary effects. Foreign output falls. The dynamics of Foreign inflation result from two opposite forces. On the one hand, in the short run, Foreign terms of trade deteriorate. Thus, because of an expenditure switching effect, Foreign households are willing to consume more Foreign goods rather than Home goods. This pushes Foreign inflation up. On the other hand, the recession makes total demand fall, which subsequently drives Foreign inflation downward.

Figure 4: IRFs to an increase in Home nominal interest rate

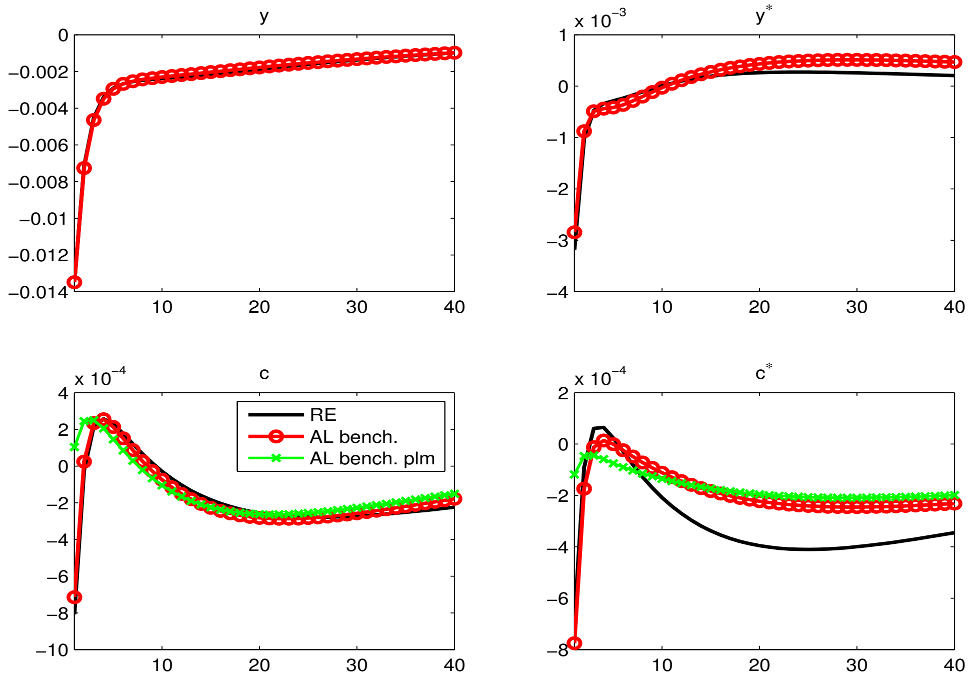


Solid line: Rational Expectations. "o" line: Adaptive learning. "+" line: Adaptive learning, $E_t[x_{t+1}]$ forecast of forward variable x based on PLM. Deviation from steady state after one-sd positive shock.

Adaptive learning under Home information bias .

Effects of Home monetary shock in the Home country. In response to a monetary shock, domestic dynamics under learning are very similar to the ones under RE. Indeed, with respect to the technological shock, there is less room for agents' "wrong" expectations to play a role. Home agents need to forecast future financial conditions in order to make current investment decisions. In a model with financial frictions, expectations on the cost of loans are a major driver of economic dynamics. The nominal interest rate is a primary element of the cost of loans. In the case of a monetary shock, the knowledge of the exogenous process of monetary innovations provides enough information to pin down the domestic interest rate. In contrast, in the case of technological shock, the forecast

Figure 5: IRFs to an increase in Home nominal interest rate



Solid line: Rational Expectations. "o" line: Adaptive learning. "+" line: Adaptive learning, $E_t[x_{t+1}]$ forecast of forward variable x based on PLM. Deviation from steady state after one-sd positive shock.

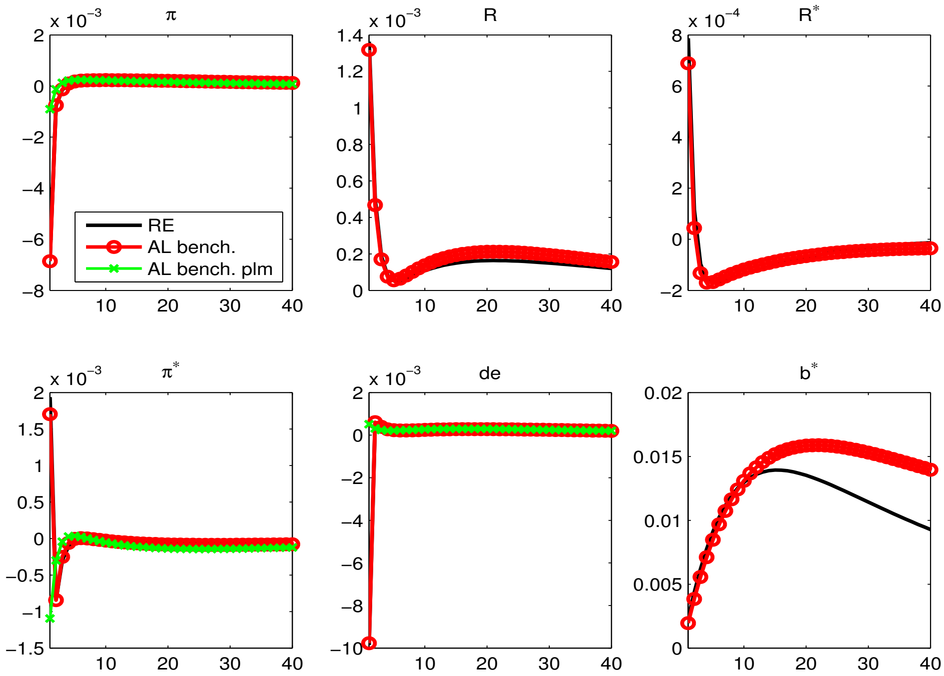
of future financial conditions is more complex as it involves forecasting output and inflation, which are general equilibrium outcomes and also depend on the extent of international financial spillovers – that are mis-perceived by Home agents.

Notice finally that, as in the case of the productivity shock, expectations after a monetary shock are more conservative than the actual evolution of the economy. This is due to the fact that agents do not consider Foreign spillovers. In particular, the expected decrease in Home inflation is smaller, as Home agents do not track the recession in the Foreign country and its feedback effects into Home.

International spillovers and departure from UIP. In response to a Home monetary shock, UIP holds. Indeed, as Home shocks are known by local agents, Home agents make little mistake in forecasting the nominal interest rate, a key variable in a model with financial frictions. The primary driver of the international interest rate differentials is the Home nominal interest rate, in the case of monetary shock. Therefore, Home agents make little mistakes in forecasting the exchange rate and UIP holds.

The shock is then transmitted to the Foreign country through terms of trade and UIP. The appreciated nominal exchange rate and its following expected depreciation – together with a higher Home interest rate – cause the Foreign interest rate to increase. As both the dynamics of the exchange rate and the ones of the Home interest rate are consistent with the RE scenario, in response to a monetary shock there are no significant deviations from UIP. The dynamics of terms of trade and the recessionary impact of the shock are then transmitted abroad. Foreign agents observe a large

Figure 6: IRFs to an increase in Home nominal interest rate



Solid line: Rational Expectations. "o" line: Adaptive learning. "+" line: Adaptive learning, $E_t[x_{t+1}]$ forecast of forward variable x based on PLM. Deviation from steady state after one-sd positive shock. de_t is the first difference of the nominal exchange rate. $de_t < 0$ is an appreciation of the Home currency.

deterioration (improvement) of Foreign (Home) terms of trade (see Fig. 11, Appendix C) but do not track the source of the shock. They expect thus a greater decrease in Foreign CPI inflation than its actual realization.

They expect also a greater real return on capital than what actually occurs. Notice also that Foreign agents are more conservative on the severity of negative spillovers and expect a smaller increase of the default threshold under learning with respect to the RE case (see Fig. 11 in Appendix C). When the latter starts decreasing (more with respect to the RE case), the arbitrage between bonds versus capital investments shifts in favor of capital. This explains why, with learning, the Foreign country experiences a greater accumulation of capital (and thus a slightly greater price of capital and net wealth), which is financed through foreign borrowing (i.e. a deterioration of their net external position) and a greater Foreign trade deficit.

Notice that, after a monetary shock, IRFs under AL and RE are very close – which is not the case after a technology shock. Indeed, depending on the source of the shock, the information the agents observe can be enough (or not) to correctly understand the transmission and forecast the evolution of the domestic economy. Following a monetary shock, Home agents know the exogenous monetary process, which provides sufficient information to pin down the expectations on the nominal interest rate and financial conditions. There is little room for expectation errors under learning in the case of the monetary shock. In contrast, agents have a hard time tracking the interest rate dynamics following a technological shock as the nominal interest rate response depends on general equilibrium outcomes (inflation and output), which in turn respond to international spillovers (that is missed by economic agents).

4.2 Quantitative results

We now focus on the quantitative predictions of our model under rational expectations. To this aim we first regress nominal exchange rate changes on the nominal interest rates differential by using simulated data:

$$log(e_{t+1}) - log(e_t) = \beta_1 + \beta_2(R_t - R_t^F) + \epsilon_t \tag{24}$$

We always find the estimated coefficient $\beta_2 = 1$, versus zero in the data (see Table 2).[16] We also provide the model's predictions when it is fed with one shock only. Whatever the shock, whether technological (line 3 in Table 2) or monetary (line 4), UIP holds (β_2 is close to 1) under rational expectations.

Table 2 displays also the model's predictions regarding the output-consumption co-movement puzzle. As in Faia (2007a, 2007b), the model under rational expectations is characterized by a strong positive output co-movement (line 2 in Table 2), which is consistent with the business cycle synchronization observed between the US and the Euro Area data since the early 1970s. However, international risk sharing is too large in the model, leading to high international correlations of consumption with greater values compared to the data.

Learning dynamics improve the model's performances along several dimensions (line 5, Table 2). First, international consumption co-movement is positive (0.29) and lies now below its predicted value under rational expectations (0.88). In particular, consumption correlation now appears much closer to the data (0.38). Second, the international correlation of consumption (0.29) is now lower than the international correlation of output (0.39), consistently with data. Third, if we regress nominal exchange rate changes on the nominal interest rate differential, we obtain a coefficient of 0.77 under learning in the Home information bias case. Given that the estimated coefficient hovers around zero in the data and around 1 under RE, we consider that our model can explain approximately 20% of departure from UIP.

We now disentangle the role of each shock, separately. It is straightforward to see that learning does not play a significant role when the economy is hit by monetary policy shocks only. The comparison of line 4 with line 7 in Table 2 shows that the conditional international correlations are very close to the RE case. Indeed, as discussed in section 4.1.2, there is very little departure from UIP (see the UIP coefficient in line 7, Table 2) and therefore the transmission to the foreign economy is little modified. When the model is hit by technology shocks only, UIP does not hold (see line 6,

[16] We do not report the constant β_1 as it is zero in the data and in all models.

Table 2: Simulations: output-consumption anomaly and departure from UIP

	Co-movements		UIP
	y,y*	c,c*	β_2
1. Data	0.53	0.38	-0.006
2. RE all shocks	0.39	0.88	1.02
3. RE tech. shocks	0.13	0.87	0.97
4. RE mon. shocks	0.50	0.93	1.10
5. AL all shocks	0.39	0.29	0.77
6. AL tech. shocks	-0.05	0.17	0.68
7. AL mon. shocks	0.49	0.88	0.99

"Data": AWM data, 1973Q1-2014Q4 for the Euro Area; US FRED for the US. See Appendix A. "AL": Home information bias model under learning. "RE": model under rational expectations. "UIP" presents the coefficient of the estimation in equation (24). "Co-movements" presents the international correlation of output and consumption. "all shocks": simulations performed with technological and monetary shocks. "tech. shocks": technological shocks only. "mon. shocks": monetary shocks only. Results are based on 1000 simulations of the model of 1000 periods' length.

Table 2). This dampens the international spillover and reduces risk sharing across countries: under AL, output and consumption co-movement are much lower than under rational expectations (see lines 3 and 6, Table 2), as discussed in Section 4.1.1. In particular, comparing lines 2 and 3, under RE, when the model is simulated with all shocks (line 2) versus with technological shocks only (line 3), we can notice that output co-movement goes down (from 0.39 with all shocks to 0.13 with technological shocks only) while consumption co-movement is barely affected (from 0.88 with all shocks versus 0.87 with technological shocks only). By comparing the model under learning and all shocks (line 5) versus the case of technological shocks only (line 6), we can notice that output co-movement goes down (from 0.39 to -0.05) and consumption co-movement is greatly reduced (from 0.29 to 0.17). With both monetary and technological shocks, as in Faia (2007*a*), the model matches the data. With learning (line 5), the unconditional international consumption synchronization is lower than the one of output and the model also features departure from UIP. This is not the case under rational expectations (line 2, Table 2), as in Faia (2007*a*).

5 Sensitivity analysis

5.1 The role of the information set

In the previous sections, we assumed a complete Home information bias. We examine now the macroeconomic consequences of relaxing this assumption. We suppose here that agents can observe more variables abroad. We choose to focus on financial variables as they are more likely to be observed worldwide and are key for the financial accelerator mechanism. We enlarge agents' information set with the other country's nominal interest rate, R, or/and price of capital, q. We proceed step by step in order to highlight the importance of the information set for the international transmission. In this section, we present IRFs after a positive Home technological shock.

In Figures 7 and 8, following a Home technological shock, we can see how more information matters for the dynamics of the Foreign economy but not much for the domestic one. This is intuitive as in the domestic country – given the domestic shock – the additional foreign information only helps tracking the feedback loop effect of the domestic shock. This is relatively small with respect to its direct effect of domestic shock on domestic variables. In contrast, the information set greatly affects the extent of international spillovers. In the Foreign country, the more information is known by agents, the more foreign decisions include the positive spillover effect induced by the financial accelerator. Responses to shocks become closer to rational expectation behaviors. This is especially the case when the interest rate of the domestic (Foreign) country is included into the Foreign (Home) information set. As discussed in section 4.1.2, the interest rate plays a great role in the international transmission. Indeed, in our model with financial frictions, the interest rate is a major driver of the cost of loans. It thus conveys much more information than other variables such as the price of capital, and helps agents track the international transmission.

The graphical intuitions are confirmed by simulation results in Table 3. Lines 4-7 in Table 3 show that the greater the information set, the closer international co-movements and UIP dynamics get to the rational expectation case. Under adaptive learning and with the full information set (line 7), the model's predictions are thus similar to results under rational expectations. Indeed, the information set is similar to the rational expectation case and agents' beliefs are initialized at rational expectations. The only difference then between rational expectations and learning lies in the update of beliefs based on the learning algorithm. Our results (line 7, Table 3) suggest that the learning algorithm alone does not quantitatively alter the magnitude of international spillovers. Therefore, international correlations and departure from UIP are the same as with rational expectations (line 2, Table 3).

5.2 Initial beliefs

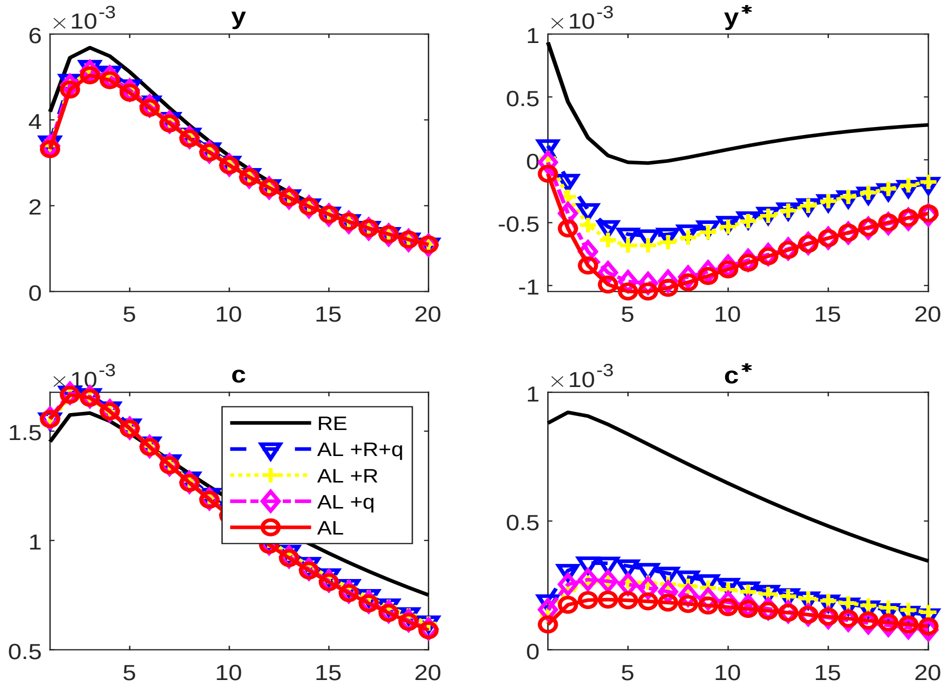
So far we have assumed that, before a shock hits the economy, variables are at their steady state values and agents' beliefs are at rational expectations. We relax this assumption. We show that if the shock hits when agents' expectations are away from the rational-expectation equilibrium, its impact is magnified (reduced) when agents are optimistic (pessimistic) at the time the shock hits the economy. This evidence is in line with Carceles-Poveda & Giannitsarou (2007) that underline the role of initial beliefs on predicted macroeconomic dynamics under learning.

Table 3: Sensitivity analysis

	Co-movements		UIP
	y,y*	c,c*	β_2
1. Data	0.53	0.38	-0.006
2. RE	0.39	0.88	1.02
3. AL	0.39	0.29	0.77
4. AL +q	0.45	0.37	0.79
5. AL +R	0.41	0.36	0.86
6. AL +R+q	0.46	0.42	0.88
7. AL +FI	0.39	0.88	1.02
8. AL+Init>RE	0.26	0.20	0.84
9. AL+Init<RE	0.49	0.36	0.75
10. AL+Gain	0.39	0.29	0.78

"UIP" presents the estimated coefficient β_2 in equation (24). "Co-movements" columns present the international correlation of output and consumption. "Data": AWM data, 1973Q1-2014Q4 for the Euro Area; US FRED for the US. See Appendix A. "RE": model under rational expectations. "AL+Gain:" Home information bias, gain at 0.05>benchmark gain. "AL+Init>RE:" Home information bias, initialization higher than RE. "AL+q:" Information set includes local variables and other country's price of capital. "AL+R:" Information set includes local variables and other country's interest rate. "AL+R+q:" Information set includes local variables and other country's interest rate and price of capital. "AL+FI:" Full Information set, it includes local variables and all foreign variable (same information set as under rational expectations). Results are based on 1000 simulations of the model of 1000 periods' length.

Figure 7: IRFs with alternative information sets. Positive Home technological shock.



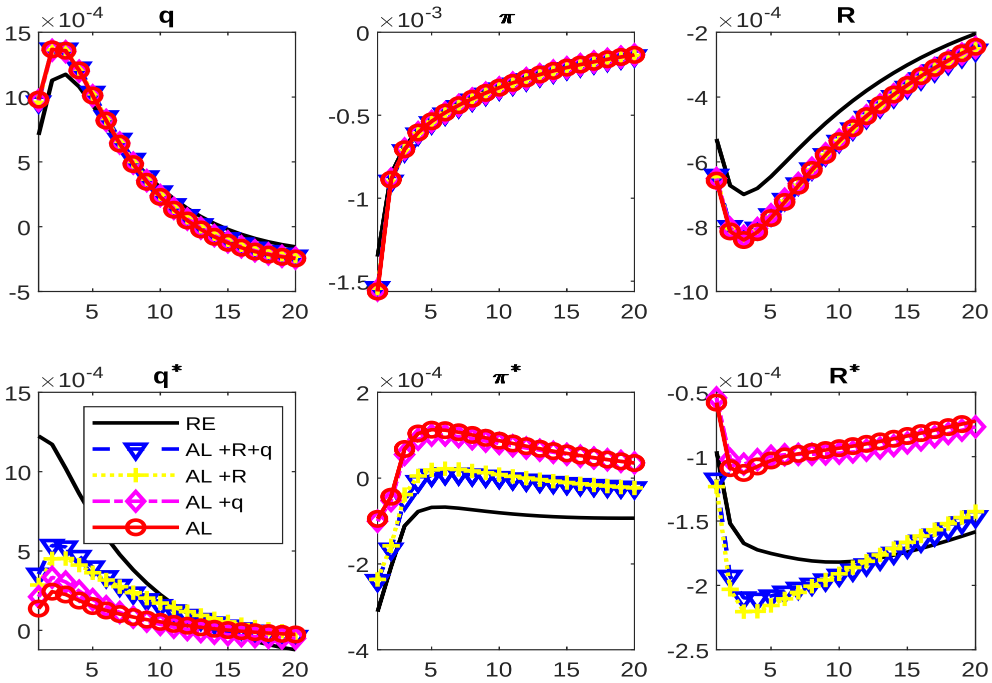
Solid line: Rational Expectations. "o" line: Adaptive learning. "diamond" line: Adaptive learning +other country's price of capital in the information set. "+" line: Adaptive learning + other country's interest rate in the information set. "triangle" line: Adaptive learning with + other country's interest rate and price of capital in the information set. Deviation from steady state after one-sd positive shock.

Figures 9 and 10 report the IRFs when learning is initialized away from rational expectations. We find that the initialization affects the amplification of Home output response to the Home technological shock. We consider here beliefs that are initialized at an higher level than under rational expectations. When agents are optimistic and the shock is pro-cyclical (positive technology shock when agents are optimistic), the response of the model is magnified.

Figure 10 clearly shows that different initial conditions matter mostly for domestic expectations and dynamics. Instead, for the Foreign dynamics, the role of the information set dominates the one of initialization in explaining the international transmission. Indeed, despite a level shift, the PLMs are still ordered following the amount of information available to the agents (the more information available the closer to rational expectation's model responses). As the contribution of our work lies in studying the international transmission under learning and imperfect information, our results on the importance of the information set remain robust to a different initialization.

Finally, Table 3 (lines 8-9) suggests that the model delivers low and positive international consumption co-movements as well as UIP departure as in the adaptive learning benchmark specification.

Figure 8: IRFs with alternative information sets. Positive Home technological shock.



Solid line: Rational Expectations. "o" line: Adaptive learning. "diamond" line: Adaptive learning +other country's price of capital in the information set. "+" line: Adaptive learning + other country's interest rate in the information set. "triangle" line: Adaptive learning with + other country's interest rate and price of capital in the information set. Deviation from steady state after one-sd positive shock.

5.3 Learning parameter

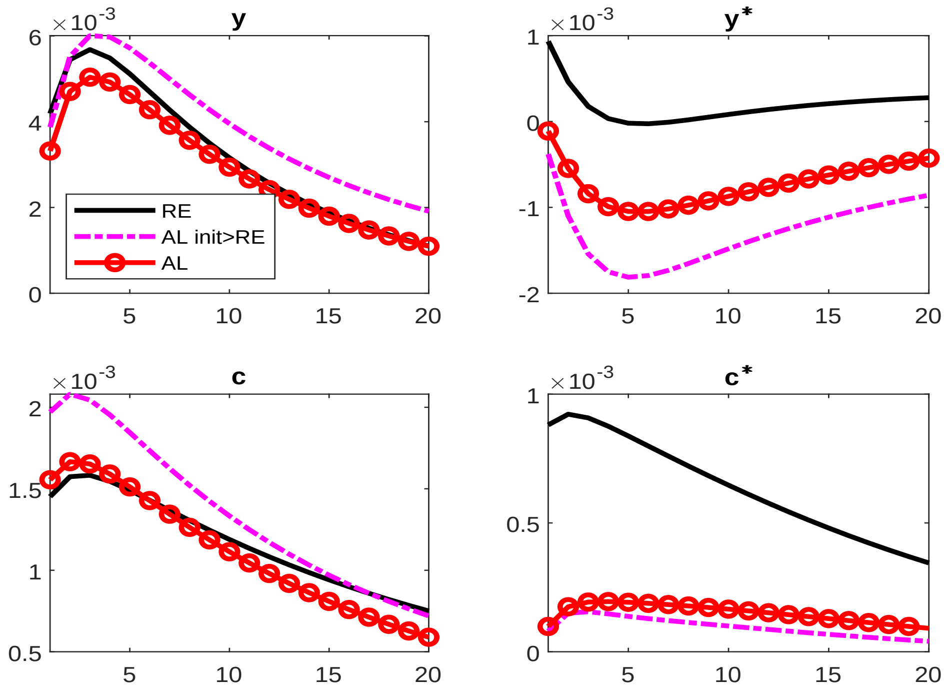
Results are also robust to different degrees of persistence of the learning algorithm. A gain coefficient of 0.05 (in the upper bound of values used in the literature) delivers very similar results to the benchmark calibration (see Table 3, line 10).

Results of this section suggest that the incompleteness of the information set quantitatively matters more than the initialization of beliefs or the persistence of the learning algorithm to explain international spillovers and departure from UIP.

6 Conclusion

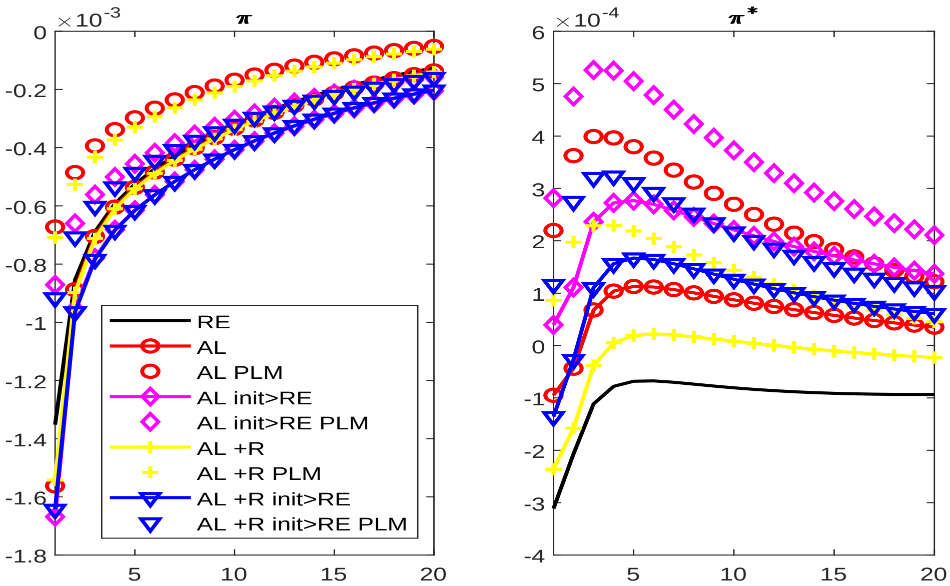
We have studied the role of imperfect information and adaptive learning for the international transmission of shocks and departure from UIP. Agents have an imperfect knowledge of their environment as their forecasts are based on complete information of the local economy but partial information of the foreign country. We show that imperfect information and learning bring the model closer to the data with a positive international co-movement of output, a low business cycle synchronization of consumption with respect to output and departure from Uncovered Interest rate Parity (UIP).

Figure 9: IRFs with alternative initializations. Positive Home technological shock.



Solid line: Rational Expectations. "o" line: Adaptive learning. "-.-" line: Adaptive learning with initial condition greater than RE. Deviation from steady state after one-sd positive shock.

Figure 10: IRFs with alternative initializations. Positive Home technological shock.



Solid line: Rational Expectations. "o" line: Adaptive learning. "diamond" line: Adaptive learning with initial condition greater than RE and "diamond" the corresponding $E_t[x_{t+1}]$. "+" line: Adaptive learning + other country's interest rate in the information set and "+" the corresponding $E_t[x_{t+1}]$. "triangle" line: Adaptive learning with initial condition greater than RE + other country's interest rate in the information set and "triangle" the corresponding $E_t[x_{t+1}]$. $E_t[x_{t+1}]$ is the forecast of variable x based on PLM. Deviation from steady state after one-sd positive shock.

Our results improve the understanding of international co-movements *and* UIP dynamics.

In this work, the amount of information available to the agents is exogenously determined. Future research could focus on the optimal choice of the information set, in the spirit of rational inattention (Sims 2003) or limited attention (Gabaix 2014), in a setting characterized by domestic and foreign information. Finally, in the light of the importance of information for aggregate dynamics, future research could re-assess the optimal mix of policies and the role of communication in order to limit information-related volatility and welfare losses.

References

Backus, D., Kehoe, P. & Kydland, F. (1992), 'International real business cycles', *Journal of Political Economy* **100**(4), 745–775.

Bae, K., Stulz, R. & Tan, H. (2008), 'Do local analysts know more? a cross-country study of the performance of local analysts and foreign analysts', *Journal of Financial Economics* **88**(3), 581–606.

Bai, Y. & Rios-Rull, J.-V. (2015), 'Demand shocks and open economy puzzles', *American Economic Review* **105**(5), 644–49.

Baxter, M. & Crucini, M. (1993), 'Explaining saving investment correlation', *American Economic Review* **83**, 416–435.

Bernanke, B., Gertler, M. & Gilchrist, S. (1999), The financial accelerator in a quantitative business cycle framework, *in* J. B. Taylor & M. Woodford, eds, 'Handbook of Macroeconomics', Vol. 1, Elsevier, pp. 1341–1393.

Carceles-Poveda, E. & Giannitsarou, C. (2007), 'Adaptive learning in practice', *Journal of Economic Dynamics and Control* **31**, 2659–2697.

Chakraborty, A. & Evans, G. (2008), 'Can perpetual learning explain the forward-premium puzzle?', *Journal of Monetary Economics* **55**, 477–490.

Christiano, L., Motto, R. & Rostagno, M. (2014), 'Risk shocks', *American Economic Review* **104**(1), 27–65.

Christiano, L., Trabandt, M. & Walentin, K. (2011), 'Introducing financial frictions and unemployment into a small open economy model', *Journal of Economic Dynamics and Control* **35**, 1999–2041.

Dvorak, T. (2005), 'Do domestic investors have an information advantage? evidence from indonesia', *Journal of Finance* **60**(2), 817–839.

Dziuda, W. & Mondria, J. (2012), 'Asymmetric information, portfolio managers, and home bias', *The Review of Financial Studies* **25**(7), 2109–2154.

Eaton, J., Kortum, S. & Neiman, B. (2016), 'Obstfeld and rogoff's international macro puzzles: a quantitative assessment', *Journal of Economic Dynamics and Control* **72**, 5–23.

Engel, C. (2016), 'Exchange rates, interest rates, and the risk premium', *American Economic Review* **106**(2), 436–474.

Eusepi, S. & Preston, B. (2018), 'The science of monetary policy: An imperfect knowledge perspective', *Journal of Economic Literature* **56**(1), 3–59.

Evans, G. W. & Honkapohja, S. (2001), *Learning and Expectations in Macroeconomics*, Princeton University Press.

Faia, E. (2007*a*), 'Finance and international business cycles', *Journal of Monetary Economics* **54**, 1018–1034.

Faia, E. (2007*b*), 'Financial differences and business cycle co-movements in a currency area', *Journal of Money, Credit and Banking* **39**(1), 151–185.

Ferreira, M., Matos, P., Pereira, J. & Pires, P. (2017), 'Do locals know better? a comparison of the performance of local and foreign institutional investors', *Journal of Banking and Finance* **82**, 151–164.

Gabaix, X. (2014), 'A sparcity-based model of bounded rationality', *Quarterly Journal of Economics* **129**(4), 1661–1710.

Gabaix, X. & Maggiori, M. (2015), 'International liquidity and exchange rate dynamics.', *Quarterly Journal of Economics* **3**, 1369–1420.

Gaspar, V., Smets, F. & Vespin, D. (2011), 'Adaptive learning, persistence, and optimal monetary policy', *Journal of the European Economic Association* **4**(2).

Gourinchas, P. & Tornell, A. (2004), 'Exchange rate puzzles and distorted beliefs', *Journal of International Economics* **64**(2), 303–333.

Hau, H. (2001), 'Location matters: An examination of trading profits', *Journal of Finance* **56**(5), 1959–1983.

Hommes, C., Massaro, D. & Weber, M. (2015), Monetary policy under behavioral expectations: Theory and experiment, Tinbergen Institute Discussion Papers 15.

Iliopulos, E., Perego, E. & Sopraseuth, T. (2016), Welfare gains for cross-country policy coordination in an open-economy banking dsge with learning., Macfinrobods wp d9.5.

Ilut, C. (2012), 'Ambiguity aversion: Implications for the uncovered interest rate parity puzzle', *American Economic Journal: Macroeconomics* **4**(3), 33–65.

Kehoe, P. & Perri, F. (2002), 'International business cycle models with endogenous incomplete markets', *Econometrica* **70**, 907–928.

Kolasa, M. & Lombardo, G. (2014), 'Financial frictions and optimal monetary policy in an open economy', *International Journal of Central Banking* **10**(1), 43–94.

Kollman, R. (2016), 'International business cycles and risk sharing with uncertainty shocks and recursive preferences', *Journal of Economic Dynamics and Control* **72**, 115–124.

Kollmann, R. (2001), 'The exchange rate in a dynamic-optimizing business cycle model with nominal rigidities : a quantitative investigation', *Journal of International Economics* **55**, 243–262.

Lewis, K. (1989), 'Changing beliefs and systematic rational forecast errors with evidence from foreign exchange', *American Economic Review* **79**(4), 621–636.

Milani, F. (2007), 'Expectations, learning and macroeconomic persistence', *Journal of Monetary Economics* **54**, 2065–2082.

Monacelli, T. (2009), 'New keynesian models, durable goods, and collateral constraints', *Journal of Monetary Economics* **56**(2).

Obstfeld, M. & Rogoff, K. (2001), The six major puzzles in international macroeconomics: Is there a common cause?, *in* B. Bernanke & K. Rogoff, eds, 'NBER Macroeconomics Annual 2000', Vol. 15, MIT PRess, pp. 339–412.

Ormeno, A. & Molnar, K. (2015), 'Using survey data of inflation expectations in the estimation of learning and rational expectations models', *Journal of Money, Credit and Banking* **47**(4), 673–699.

Orphanides, A. & Williams, J. (2005), 'Inflation scares and forecast-based monetary policy', *Review of Economic Dynamics* pp. 498–527.

Orphanides, A. & Williams, J. (2007), 'Robust monetary policy with imperfect knowledge', *Journal of Monetary Economics* **54**, 1406–1435.

Orphanides, A. & Williams, J. (2008), 'Learning, expectations formation, and the pitfalls of optimal control monetary policy', *Journal of Monetary Economics* **55**, S80–S96.

Pintus, P. & Suda, J. (2018), 'Learning financial shocks and the great recession', *Review of Economic Dynamics* .

Rychalovska, Y., Slobodyan, S. & Wouters, R. (2016), Financial frictions, expectations and business cycle: Evidence from an estimated dsge model, Mimeo.

Schmitt-Grohe, S. & Uribe, M. (2003), 'Closing small open economy models', *Journal of International Economics* **61**(1), 163–185.

Sims, C. (1980), 'Macroeconomics and reality', *Econometrica* **48**(1), 1–48.

Sims, C. (2003), 'Implications of rational inattention', *Journal of Monetary Economics* **50**, 665–690.

Slobodyan, S. & Wouters, R. (2012*a*), 'Learning in a medium-scale dsge model with expectations based on small forecasting models', *American Economic Journal: Macroeconomics* **4**(2), 65–101.

Slobodyan, S. & Wouters, R. (2012*b*), 'Learning in an estimated medium-scale dsge model', *Journal of Economic Dynamics and Control* **36**, 26–46.

Teo, M. (2009), 'The geography of hedge funds', *Review of Financial Studies* **22**(9), 3531–3561.

Tille, C. & van Wincoop, E. (2014), 'International capital flows under dispersed private information', *Journal of International Economics* **93**(1), 31–49.

Van Nieuwerburgh, S. & Veldkamp, L. (2009), 'Information immobility and the home bias puzzle', *Journal of Finance* **64**(3), 1187–1215.

W.A., B. & Evans, G. (2006), 'A simple recursive forecasting model', *Economics Letters* **91**, 158–166.

Appendix

A Data

US. Quarterly US data is downloaded from FRED Economic data, from the Federal Reserve bank of Saint Louis. GDP is Real Gross Domestic Product, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate. Consumption is Real Personal Consumption Expenditures, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate. Nominal interest rate used in equation (24) is 3-Month Treasury Bill: Secondary Market Rate, Percent, Quarterly, Not Seasonally Adjusted.

Euro Area. EA data comes from AWM (Area Wide Model) dataset, available online from the Euro Area Business Cycle Network website. We use the following quarterly time series: real GDP, Real private consumption, Nominal exchange rate (euro per USD), Nominal shirt-term interest rates.

B Model equations

In our numerical analysis we do consider the following instantaneous utility function for households in both countries:

$$\begin{aligned} U_t &= \frac{C_t^{1-\sigma}}{1-\sigma} + \Psi \ln(1-N_t) \\ U_t^* &= \frac{C_t^{*1-\sigma}}{1-\sigma} + \Psi \ln(1-N_t^*) \end{aligned}$$

where σ denotes the intertemporal elasticity of substitution and Ψ is the parameter associated to leisure. Marginal utilities of consumption and labor, respectively, are thus:

$$\begin{array}{c} U'_{ct} = C_t^{-\sigma}\ , \quad U'_{Nt} = -\Psi\frac{1}{1-N_t} \\ U'^*_{ct} = C_t^{*-\sigma}\ , \quad U'^*_{Nt} = -\Psi\frac{1}{1-N_t^*} \end{array}$$

The model consists in the following list of equilibrium equations.

$$X_{Ht} = (1-\gamma)\left[f_t\right]^{-\eta} X_t^c + \left[tot_t f_t^*\right]^{-\eta} \gamma^* X_t^{c*} + \frac{\omega_P}{2}\left(\pi_{Ht}-1\right)^2 \tag{1.}$$

$$X_{Ft}^* = (1-\gamma^*)\left[f_t^*\right]^{-\eta} X_t^{*c} + \left[\frac{f_t}{tot_t}\right]^{-\eta} \gamma X_t^c + \frac{\omega_P}{2}\left(\pi_{Ft}^*-1\right)^2 \tag{2.}$$

$$X_t^c = C_t^c + I_t + (1-\varsigma_t)\left[1-\Gamma\left(\tilde{\omega}_t\right)\right]\mathcal{R}_t^k \frac{Q_{t-1}}{\pi_t}K_{t-1} + \mu G\left(\tilde{\omega}_t\right) R_t^k \frac{Q_{t-1}}{\pi_t}K_{t-1} + \Phi\left(\frac{I_t}{K_{t-1}}\right)K_{t-1} \quad (3.)$$

$$X_t^{*c} = C_t^{*c} + I_t^* + (1-\varsigma_t^*)\left[1-\Gamma^*\left(\tilde{\omega}_t^*\right)\right]\mathcal{R}_t^{*k} \frac{Q_{t-1}^*}{\pi_t^*}K_{t-1}^* + \mu G^*\left(\tilde{\omega}_t^*\right) R_t^{*k} \frac{Q_{t-1}^*}{\pi_t^*}K_{t-1}^* + \Phi\left(\frac{I_t^*}{K_{t-1}^*}\right)K_{t-1}^* \quad (4.)$$

$$K_t = (1-\delta)K_{t-1} + I_t \quad (5.)$$

$$K_t^* = (1-\delta)K_{t-1}^* + I_t^* \quad (6.)$$

$$C_t^{-\sigma} = \beta R_t E_t\left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}\right] \quad (7.)$$

$$C_t^{*-\sigma} = \beta R_t^* E_t\left[\frac{C_{t+1}^{*-\sigma}}{\pi_{t+1}}\right] \quad (8.)$$

$$E_t\frac{R_{t+1}^k}{R_t} = E_t\frac{1}{\frac{\left[1-\Gamma(\tilde{\omega}_{t+1})\right]\left[\Gamma'(\tilde{\omega}_{t+1})-\mu G'(\tilde{\omega}_{t+1})\right]}{\Gamma'(\tilde{\omega}_{t+1})} + \left[\Gamma\left(\tilde{\omega}_{t+1}\right) - \mu G\left(\tilde{\omega}_{t+1}\right)\right]} \quad (9.)$$

$$E_t\frac{R_{t+1}^{*k}}{R_t^*} = E_t\frac{1}{\frac{\left[1-\Gamma^*\left(\tilde{\omega}_{t+1}^*\right)\right]\left[\Gamma'^*\left(\tilde{\omega}_{t+1}^*\right)-\mu G'^*\left(\tilde{\omega}_{t+1}^*\right)\right]}{\Gamma'^*\left(\tilde{\omega}_{t+1}^*\right)} + \left[\Gamma^*\left(\tilde{\omega}_{t+1}^*\right) - \mu G^*\left(\tilde{\omega}_{t+1}^*\right)\right]} \quad (10.)$$

$$R_t^k = \pi_t\frac{f_t\frac{\alpha\frac{Y_t}{K_{t-1}}}{X_t} + \varpi\left(\frac{I_t}{K_{t-1}}-\delta\right)\frac{I_t}{K_{t-1}} - \frac{\varpi}{2}\left(\frac{I_t}{K_{t-1}}-\delta\right)^2 + Q_t(1-\delta)}{Q_{t-1}} \quad (11.)$$

$$R_t^{*k} = \frac{\pi_t^*}{Q_{t-1}^*}\left[\frac{\alpha\frac{Y_t^*}{K_{t-1}^*}}{X_t^*}f_t^* + \varpi\left(\frac{I_t^*}{K_{t-1}^*}-\delta\right)\frac{I_t^*}{K_{t-1}^*} - \frac{\varpi}{2}\left(\frac{I_t^*}{K_{t-1}^*}-\delta\right)^2 + Q_t^*(1-\delta)\right] \quad (12.)$$

$$Q_{t-1}K_{t-1}\left[1-\left[\Gamma\left(\tilde{\omega}_t\right)-\mu G\left(\tilde{\omega}_t\right)\right]\frac{R_t^k}{R_{t-1}}\right] = nw_{t-1} \quad (13.)$$

$$Q_{t-1}^*K_{t-1}^*\left[1-\left[\Gamma\left(\tilde{\omega}_t^*\right)-\mu G\left(\tilde{\omega}_t^*\right)\right]\frac{R_t^{*k}}{R_{t-1}^*}\right] = nw_{t-1}^* \quad (14.)$$

$$nw_t = \varsigma\left[1-\Gamma\left(\tilde{\omega}_t\right)\right]\frac{R_t^k}{\pi_t}Q_{t-1}K_{t-1} \quad (15.)$$

$$nw_t^* = \varsigma_t\left[1-\Gamma\left(\tilde{\omega}_t^*\right)\right]\frac{R_t^{*k}}{\pi_t}Q_{t-1}^*K_{t-1}^* \quad (16.)$$

$$Y_t = A_t K_{t-1}^{\alpha} N_t^{1-\alpha} \tag{17.}$$

$$Y_t^* = A_t^* K_{t-1}^{*\alpha} N_t^{*1-\alpha} \tag{18.}$$

$$C_t^{-\sigma} f_t \frac{(1-\alpha)\frac{Y_t}{N_t}}{X_t}(1-N_t) = \Psi \tag{19.}$$

$$C_t^{*-\sigma} f_t^* \frac{(1-\alpha)\frac{Y_t^*}{N_t^*}}{X_t^*}(1-N_t^*) = \Psi \tag{20.}$$

$$\omega_P(\pi_{Ht}-1)\pi_{Ht} = X_{Ht}\upsilon\left[\frac{(1-\upsilon)}{\upsilon} + \frac{1}{X_t}\right] + \beta\omega_P E_t \frac{U'_{ct+1}}{U'_{ct}}(\pi_{Ht+1}-1)\frac{f_{t+1}}{f_t}\pi_{Ht+1} \tag{21.}$$

$$\omega_P(\pi_{Ft}^*-1)\pi_{Ft}^* = X_{Ft}^*\upsilon\left[\frac{(1-\upsilon)}{\upsilon} + \frac{1}{X_t^*}\right] + \beta\omega_P E_t \frac{U^{*\prime}_{ct+1}}{U^{*\prime}_{ct}}\left(\pi_{Ft+1}^*-1\right)\frac{f_{t+1}^*}{f_t^*}\pi_{Ft+1}^* \tag{22.}$$

$$R_t^n = \left(R_{t-1}^n\right)^{\chi}\left(\bar{R}^n\left(\frac{\pi_t}{\bar{\pi}}\right)^{b_\pi}\left(\frac{y_t}{y}\right)^{b_y}\right)^{1-\chi} mp_t \tag{23.}$$

$$R_t^{n*} = \left(R_{t-1}^{n*}\right)^{\chi}\left(\bar{R}^{*n}\left(\frac{\pi_t^*}{\bar{\pi}^*}\right)^{b_\pi}\left(\frac{y_t^*}{y^*}\right)^{b_y}\right)^{1-\chi} mp_t^* \tag{24.}$$

$$Q_t = \left[1 + \varpi\left(\frac{I_t}{K_{t-1}} - \delta\right)\right] \tag{29.}$$

$$Q_t^* = \left[1 + \varpi\left(\frac{I_t^*}{K_{t-1}^*} - \delta\right)\right] \tag{26.}$$

$$1 = \beta\left[R_t^* - \zeta\left(e^{b_t^*-b^*} - 1\right)\right] E_t\left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\frac{e_{t+1}}{\pi_{t+1}e_t}\right] \tag{27.}$$

$$f_t = \left[(1-\gamma) + \gamma\left(\frac{1}{tot_t}\right)^{1-\eta}\right]^{\frac{-1}{1-\eta}} \tag{28.}$$

$$f_t^* = \left[(1-\gamma) + \gamma tot_t^{1-\eta}\right]^{\frac{-1}{1-\eta}} \tag{29.}$$

$$\frac{tot_t}{tot_{t-1}} = \frac{\pi_{Ht}}{\frac{e_t}{e_{t-1}}\pi_{Ft}^*} \tag{30.}$$

$$f_t = f_{t-1}\frac{\pi_{Ht}}{\pi_t^c} \tag{31.}$$

$$f_t^* = f_{t-1}^*\frac{\pi_{Ft}^*}{\pi_t^{c*}} \tag{32.}$$

$$X_{Ht} = Y_t \tag{33.}$$

$$X_{Ft}^* = Y_t^* \tag{34.}$$

$$R_t^F = R_t^* - p\left(b_t^*\right) \tag{35.}$$

$$b_t^* - \frac{b_{t-1}^*}{\pi_t}\frac{e_t}{e_{t-1}} = \left(R_{t-1}^F - 1\right)\frac{b_{t-1}^*}{\pi_t}\frac{e_t}{e_{t-1}} + f_t Y_t - X_t^c \tag{36.}$$

$$ca_t = b_t^* - \frac{b_{t-1}^*}{\pi_t}\frac{e_t}{e_{t-1}} \tag{37.}$$

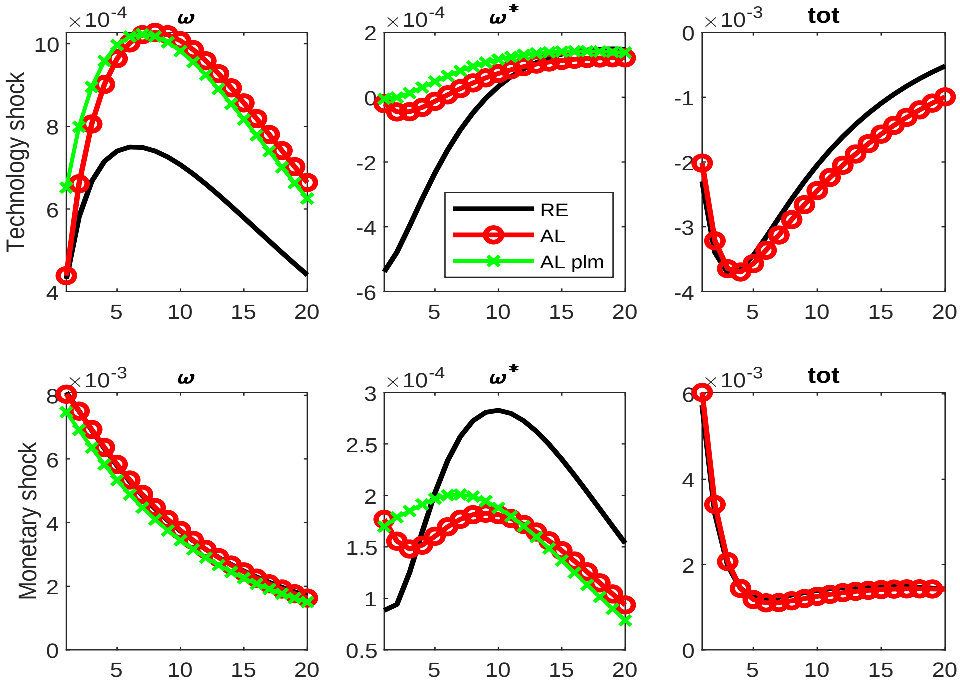
$$tb_t = f_t Y_t - X_t^c \tag{38.}$$

The model also includes the law of motions of the technology and monetary shocks:

$$\begin{aligned}
\log A_t &= \rho_A \log A_{t-1} + \varepsilon_t^{as} \\
log A_t^* &= \rho_A\, log(A_{t-1}^*) + \varepsilon_t^{as*} \\
\log mp_t &= \rho_{mp} \log mp_{t-1} + \varepsilon_t^{mp} \\
\log mp_t^* &= \rho_{mp} \log mp_{t-1}^* + \varepsilon_t^{mp*}
\end{aligned}$$

C Additional IRFs

Figure 11: IRFs: Terms of trade (tot, Home price relative to Foreign price) and default threshold at Home ω and abroad ω^*



Top panel: "Technology shock" positive Home technology shock. Bottom panel "Monetary shock" positive monetary shock. Variable tot is Home terms of trade defined as the ratio of the price of domestic goods over the price of foreign goods $tot_t = \frac{P_{Ht}}{e_t P^*_{Ft}}$. Solid line: Rational Expectations. "o" line: Adaptive learning. $E_t[x_{t+1}]$ is the forecast of variable x based on PLM. Deviation from steady state after one-sd positive shock.